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Electromagnetic and Weak Interactions in Stochastic Space-Time: A Review

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A scheme for a nonlocal theory of quantized fields based on the hypothesis of stochastic space is proposed. Within this scheme the gauge-invariant quantum electrodynamics of particles with spin 0, 1/2, 1 and four-fermion weak interactions are constructed, and nonlocal corrections to the anomalous magnetic moments of leptons and to the Lamb shift are calculated. Some consequences of the neutrino oscillations and the electromagnetic properties of neutrinos are considered in detail. Further the rare decay $K_L^0 \to \mu^+\mu^-$ and the mass difference of K_L^0 and K_S^0 mesons are investigated in this model. It is shown that the parameter of nonlocality (elementary length l) of weak interactions which can characterize a domain of unification of weak and electromagnetic interactions is $\sim 10^{-16}$ cm. The low-energy experiments imply that quantum electrodynamics is valid up to distances of order $\sim 10^{-15}$ cm.

1. INTRODUCTION

One of the fundamental principles of the quantum field theory (QFT) is the locality condition (the commutation rules). More clearly, this means that the commutator of operators of physical fields disappears outside the light cone. On the other hand, this property of locality ensures independence of events separated by spacelike intervals, i.e., the causality condition is space-time (usually called the microcausality). A strict formulation of the microcausality in QFT was given by Bogolubov and coworkers (1959).

A possible violation of locality at small distances is conditioned by intrinsic problems of QFT like the ultraviolet divergences, the problem of

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electron self-energy, etc. This problem appears especially inevitable as soon as we want to describe extended objects within the QFT.

Among attempts at a self-consistent construction of QFT with a reasonable locality (macrocausality) condition at small distances, a distinguished role is played by nonlocal QFTs. There exist two different approaches to construct such a theory. Supporters of the first approach assume that in Nature there exists some new fundamental constant of dimension of length, together with such constants as the velocity of light c and the Planck constant $\hbar$. Further, they assume that at distances of this new universal length l one must expect a principal change in our concepts of the physical world and in particular the concept of space(-time) and locality (causality). The main problems of this approach and possible ways of changing the contemporary theory were discussed by Efimov (1977) and Kadyshevsky (1980), where earlier references concerning this problem are cited. Some possibilities of introducing the concept of fundamental length in physics were considered by Blokhintzev (1973), Cheon (1978, 1979), Brout et al. (1980), Ginzburg (1975), Ehrlich (1978), Fubini (1974), Hsu and Mac (1979), Markov (1958), Lacroix (1979), Takano (1961, 1967), and Yukawa (1950).

The second approach is based on the assumption that the parameter l is not a universal fundamental constant but characterizes only the domain of nonlocal interaction of the considered quantum fields (see, for example, Efimov, 1977). Thus the parameter l of dimension of length arises inevitably at any attempt to introduce nonlocality into the theory. Recently, high-precision measurements (Bailey et al., 1979; Van Dyck et al., 1977; Robiscoe, 1968; Robiscoe and Shyn, 1970) in atomic physics, for example, measurements of the anomalous magnetic moment (AMM) of muons (electrons) and of the Lamb shift, have given the following restrictions on the parameter l: $l \lesssim 10^{-15}$ cm ($l \lesssim 10^{-13}$ cm) and $l \lesssim 10^{-13}$ cm, respectively (see Efimov, 1977; Hsu and Mac, 1979) [for discussion of various theoretical contributions, where earlier references can be found, as well as for a review comparing the theory and experiment, see Brodsky and Drell (1970), Lautrup et al. (1972); Calmet et al. (1977), and Kinoshita (1979)]. From the high-energy experimental data it follows that $l \lesssim 10^{-16}$ cm (Flügge, 1980; Wolf, 1980; Beron et al., 1978; Barber et al., 1979; Bartel et al., 1980; Berger et al., 1980).

Tests of locality are usually performed by modification of particle propagators and vertex functions. In particular, tests of quantum electrodynamics are carried out in terms of modified electron and photon propagators (see Kraus, 1975; Magg et al., 1972, and Ringhofer and Salecker, 1980). On the other hand, it is well known that the introduction of nonlocality into theories which belong to the second approach leads to a change of particle propagators (Efimov, 1977). However, this change is not arbitrary and is determined by fundamental principles of QFT like Lorentz covariance, finiteness, unitarity, causality, and gauge invariance. Roughly speaking, the aim of a nonlocal theory is to find restrictions on the choice of a form for nonlocal particle propagators (or so-called form factors of the theory).

Thus an analysis of experimental data for testing locality must be performed within a theory that satisfies the basic principles of QFT. The problem of constructing a nonlocal theory satisfying the above-mentioned basic principles is now solved successfully (Efimov, 1972, 1977). One assumes in this theory that neutral particles (for example, photons and neutrinos) are "carriers" of nonlocality, while the charged fields are considered to be local.

Thus introduction of the nonlocality into this theory leads to changing only the propagator of the uncharged particles in the perturbation series for the S matrix satisfying all the general requirements: causality, unitarity, gauge invariance, etc. (Efimov, 1977). For example,

$$g_{\mu\nu}/(-k^2-i\varepsilon)\to g_{\mu\nu}V_A(-k^2l_A^2)/(-k^2-i\varepsilon)$$

for photons, and

$$\hat{k}/(-k^2-i\varepsilon)\to \hat{k}V_\nu(-k^2l_\nu^2)/(-k^2-i\varepsilon)$$

for neutrinos, where $V_{A,\nu}(z)$ are entire functions of a finite order of growth $\rho \geqslant 1/2$ in the complex z plane which decrease rapidly enough when $z=p^2\to-\infty$ (in the Euclidean direction), and l_A and l_ν characterize the size of a domain where electromagnetic and weak interactions become nonlocal.

However, the above-mentioned way of introducing nonlocality into the theory does not remove all ultraviolet divergences from the perturbation series for the S matrix. There are divergences in the so-called vacuum polarization diagrams constructed using propagators of the charged particles. Usually, in order to remove the divergences in these diagrams, one uses the modified Pauli–Villars regularization (see Bogolubov and Shirkov, 1959; Slavnov, 1974; Efimov, 1972). This method of regularization of singular functions is to be understood as a formal procedure only and has no definite physical meaning.

It is generally accepted at present that the essence of mathematical methods for removing divergences is in a more or less explicit way connected with the properties of space at small distances or with the very nature of high-energy interactions which is inherent to all types of interactions. Thus we believe that the method of eliminating ultraviolet divergences must be the same for all types of diagrams and must have a clear physical meaning.

Some attempts have been undertaken to construct quantum field theory in a stochastic space (see Markov, 1959; Takano, 1961, 1967; Ingraham, 1967 and references therein).

A stochastic space that can be used in theories of elementary particles was first considered by March (1934, 1937), Markov (1958), and Yukawa (1966) (see also review of Blokhintsev, 1975). Mathematical spaces with a stochastic metric and a quantized domain were investigated by Frederick (1976) and Roy (1979), respectively. Prugovecki's (1978a, b) papers are devoted to the construction of relativistic kinematics for massive and massless particles in the stochastic phase space. An original idea of this review, i.e., construction of the theory of electromagnetic and weak interactions of leptons within the framework of a stochastic space, was first formulated by Dineykhan and Namsrai (1977, 1978).

Papers by Namsrai (1980a, b) are devoted to investigation of the stochastic space $R_4(\hat{x})$ with

$$\hat{x}=(x_0+ib_4,\mathbf{x}+\mathbf{b}) \qquad (x_0=ct) \tag{1}$$

$x=(x_0,\mathbf{x})$ being the regular part of the components $\hat{x}$ and $b^E=(b_4,\mathbf{b})$ some small random vector with a distribution $w(b_E^2/l^2)$ obeying the conditions

$$\int dw\left(b_E^2/l^2\right)=1, \qquad dw\left(b_E^2/l^2\right)\geqslant 0 \tag{2}$$

here l is some universal length (a scale of errors). In our case, the universal length l characterizes physically a certain domain within which the existing space concepts and causality conditions may be violated and the stochastic properties or fluctuations in the metrics can be manifested if they exist.

Dynamics of particles (Namsrai, 1980a, b), relativistic Feynman-type integrals (Namsrai, 1980c), and Euclidean Markov field (Namsrai 1981a) have been investigated within the framework of the stochastic space $R_4(\hat{x})$ (see also review of Namsrai, 1981b). It appears that a field obtained by averaging in the space $R_4(\hat{x})$ turns out to be the nonlocal field considered by Efimov (1968, 1977) (see section 2). Equivalence of these approaches leads to the following hypothesis: The origin of the form factors which change the electromagnetic (Efimov, 1972) and weak (Efimov et al., 1973) potentials at small distances and the properties of the vacuum polarization may be connected with the stochasticity of space on the microscale.

Thus within the framework of our scheme, all fields, both neutral and charged ones, become spread out (nonlocalized) over the space. This allows one to take into account in a unified way an effect of stochasticity (or nonlocality) in all physical processes. However, the change of charged particle propagators essentially complicates the proof of gauge invariance of the theory.

This paper is a review of the construction of gauge-invariant quantum electrodynamics for particles with spins 1/2, 0, and 1 and of the four-fermion theory of weak interactions in stochastic space. Within this framework, the electromagnetic and weak processes are investigated and contributions to the AMM of leptons and to the Lamb shift due to nonlocality (stochasticity) are estimated. The problem of neutrino oscillations and its consequences are also discussed in this theory. Here the considerations are mainly concerned with the low-energy processes. Notice that at very high energies testing of locality of the theory may be difficult because of interference effects between the electromagnetic and weak interactions. For example, in the standard model of electroweak interactions, testing of QED will be disturbed by the interference with the weak effects due to Z^0 bosons.

2. STOCHASTIC SPACE AND NONLOCALITY

In the relativistic theory the space in which the physical processes are investigated is the Minkowski space. Now the problem appears as to how to introduce the stochasticity into this space.

Indefiniteness of the metrics of this space leads to specific problems which do not appear in the case of Euclidean space. These specific difficulties in the physical space are connected with the invariance assumption and normalization condition for the probability of an interval in the indefinite-metric space (see Blokhintsev, 1973, for details). For example, the requirement of invariance, roughly speaking, means that the distribution $w(b_\mu)$ of the vector b_μ must be a function of the interval $b^2 = b_\mu b^\mu = b_0^2 - \mathbf{b}^2$, and the normalization condition gives the equality

$$\int dw(b_\mu b^\mu) = 1$$

These two conditions cannot be, in fact, fulfilled simultaneously in the Minkowski space. It turns out that one can get rid of the above-mentioned difficulties by making the following assumptions (Namsrai, 1980a,c).

1. The physical quantities are considered as functions of complex times $t + i\tau$ in the limit $\tau \to 0$.

2. The space stochasticity appears in the Euclidean space $(\tau, \mathbf{x})$ but not in the Minkowski space $(t, \mathbf{x})$. The importance of the method of shift $x_0 \to x_0 + i\tau$ in the time variable in quantum field theory and quantum mechanics was noted by Alebastrov and Efimov (1974) and Davidson (1978), respectively (see Efimov, 1977, also). Thus in our model the actual points of the space $R_4(\hat{x})$ consist of two parts (1), where $b_4 = \tau$, and any physical quantity f in $R_4(\hat{x})$ depends on arguments of the type $x_0 + ib_4$, $\mathbf{x}+\mathbf{b}$, i.e., $f = f(x_0 + ib_4, \mathbf{x}+\mathbf{b})$.

Since, in our model the actual points of the space are of a stochastic nature, these points cannot be used as a basis for a coordinate system, nor can one take a derivative with respect to them. However, the space of common experience (i.e., the laboratory frame) is nonstochastic on a large scale. It is only in the microworld where the stochasticity manifests itself. One can then continue mathematically from the microworld to this large-scale nonstochastic space. This mathematical construction provides a nonstochastic space to which the stochastic physical space can be referred. This is the Frederick (1976) argument. In our case the mathematical construction reduces to averaging with the distribution $w(b_E^2/l^2)$ at any point of the space $R_4(\hat{x})$ at a given time.

Therefore the averaged quantity $\langle f(\hat{x})\rangle$ on $R_4(\hat{x})$ with $w(b_E^2/l^2)$ is called the physical value of $f(\mathbf{x}, t)$ (Namsrai, 1980b). Especially, the considered field $\varphi(\hat{x})$ after averaging in $R_4(\hat{x})$ acquires the following form:

$$\varphi_R(x) \equiv \langle \varphi(\hat{x})\rangle_R = \int d^4b_E\, w\left(b_E^2/l^2\right)\varphi(x_0 + ib_4, \mathbf{x}+\mathbf{b}) \tag{3}$$

This is just the nonlocal object which has been carefully investigated by Efimov (1968, 1977) from the viewpoint of the distributions

$$K(x) = \sum_{n=0}^{\infty} c_n \Box^{2n}\delta^4(x), \qquad \Box = -\frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial \mathbf{x}^2} \tag{4}$$

The space-time properties of these depend essentially on the sequence of coefficients c_n. Efimov has shown that the objects $\varphi_R(x)$ constructed by these distributions are spread out (nonlocalized) over space. Thus the relativistic invariant distributions $K(x)$ give a correct description of extended objects. In this case, roughly speaking, the parameter l may be identified with the size of an extended object (a particle).

Due to Efimov (1977) we can calculate the causal Green functions of the spread-out field $\varphi_R(x)$ [keeping in mind that the T-ordering symbol concerns the field $\varphi(\hat{x})$, where $\varphi(\hat{x})$ is a scalar field with a mass m] by the following formula

$$\mathcal{D}(x_1 - x_2) = \langle 0|T(\varphi_R(x_1)\varphi_R(x_2))|0\rangle$$

$$= \iint d^4b_{1E}\, d^4b_{2E}\, w(b_{1E}^2/l^2) w(b_{2E}^2/l^2)$$

$$\times \langle 0|T(\varphi(x_{10}+ib_{14},\mathbf{x}_1+\mathbf{b}_1)\varphi(x_{20}+ib_{24},\mathbf{x}_2+\mathbf{b}_2))|0\rangle$$

$$= \iint d^4b_{1E}\, d^4b_{2E} w(b_{1E}^2/l^2) w(b_{2E}^2/l^2) \frac{1}{i} \int \frac{d^4p}{(2\pi)^4}$$

$$\times \frac{\exp[ip_0(x_{10}-x_{20}+ib_{14}-ib_{24}) - i\mathbf{p}(\mathbf{x}_1-\mathbf{x}_2+\mathbf{b}_1-\mathbf{b}_2)]}{m^2-p^2-i\varepsilon}$$

$$= \frac{1}{i}\int \frac{d^4p}{(2\pi)^4} \frac{|\tilde{K}(-p^2l^2)|^2}{m^2-p^2-i\varepsilon} \exp[ip(x_1-x_2)] \tag{5}$$

where

$$\tilde{K}(-p^2l^2) = \int d^4b_E \exp(-i\mathbf{p}\mathbf{b} - p_0b_4) w(b_E^2/l^2)$$

$$= (2\pi)^2 \int_0^\infty d\rho\, \rho^2 w(\rho^2/l^2) \mathcal{J}_1\left[\rho(-p^2)^{1/2}\right] \Big/ (-p^2)^{1/2}$$

$$p^2 = p_0^2 - \mathbf{p}^2 \tag{6}$$

Here $\mathcal{J}_1(z)$ is the Bessel function.

Notice that the Fourier transforms of distributions (4) are determined by a representation of the type (6). The expression (6) is investigated as usually in the Euclidean domain $p^2<0$ of the momentum variable p (Efimov, 1968, 1977). The passage to the case of $p^2>0$ is done by an analytic continuation (see Efimov, 1968, 1977). Further we are interested only in the class of distributions $w(b_E^2/l^2)$ for which $\tilde{K}(z)$ (6) are entire functions of the variable z with a finite order of growth $\infty > \rho \geqslant 1/2$ and which decrease rapidly enough when $z = p^2 \to -\infty$ (in the Euclidean direction).

So, starting with the hypothesis of stochastic space we come to the nonlocal theory of Efimov with the only difference that the causal Green function $S(\hat{p})$ of any charged particle is replaced by

$$S(\hat{p}) \to S_R(\hat{p}) = V(-p^2l^2) S(\hat{p})$$

where

$$V(-p^2l^2) = |\tilde{K}(-p^2l^2)|^2, \qquad \hat{p} = p_\mu \gamma_\mu$$

for which the Mellin representation

$$V(-p^2l^2)=\frac{1}{2i}\int_{-\beta+i\infty\ (0<\beta<1)}^{-\beta-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}l^{2\zeta}(m^2-p^2-i\varepsilon)^{\zeta} \tag{7}$$

is valid. The form of the functions $V(-p^2l^2)$ and $v(\zeta)$ depends on the form of the distribution $w(b_E^2/l^2)$. For example (Namsrai, 1981a), let

$$w_m(y^2)=\begin{cases}\left(\frac{\pi}{2}ml\right)^{1/2}\pi^{-2}l^{-4}\frac{(ml)^2(1-y^2)^{-1/4}}{4(\sin ml/ml-\cos ml)} & \\ \times\mathcal{J}_{-1/2}\left(ml(1-y^2)^{1/2}\right), & 0\leqslant y<1\\ 0 & y\geqslant 1\end{cases}$$

Then we obtain

$$V_s=|\tilde{K}_m(-p^2l^2)|^2=\left(\frac{m^2l^2}{\sin ml/ml-\cos ml}\right)^2[(m^2-p^2)l^2]^{-2}$$

$$\times\left\{\sin[(m^2-p^2)l^2]^{1/2}/[(m^2-p^2)l^2]^{1/2}-\cos[(m^2-p^2)l^2]^{1/2}\right\}^2 \tag{8}$$

$$v_s(\zeta)=92^{4+2\zeta}[2\zeta^2+7\zeta+5]/\Gamma(7+2\zeta) \tag{9}$$

Here m is some parameter ($m^2l^2\ll 1$) which can be identified with the particle mass. Notice that the form factor (8) in the case $m=0$ describes the spread-out electron as a uniformly charged sphere of radius l (Efimov, 1977).

The main restrictions in the choice of form factors $V(-p^2l^2)$ as entire functions arise from the fundamental theoretical principles, i.e., from unitarity and causality (Alebastrov and Efimov, 1973, 1974).

The physical meaning of form factors consists of changing the form of potentials between interacting fields (for example, the Coulomb and Yukawa laws) at small distances and in making the theory finite in each order of the perturbation series of the theory in coupling constant (Efimov, 1977). The question about a possible unique choice of the form factors [in our case of distributions $w(b_E^2/l^2)$] was discussed by Efimov (1977) (see also Papp, 1975).

3. GENERALIZATION OF KROLL'S PROCEDURE AND GAUGE INVARIANCE OF THE THEORY

The hypothesis of stochastic space leads thus to a change of propagators of both neutral and charged particles. It is well known that any modification of the local causal Green functions of charged particles results in a violation of some algebraical relations (for example, the Ward–Takahashi identities). The fulfilment of these algebraical relations grants the gauge invariance of the theory. There are numerous papers (Kroll, 1966; Kraus, 1975; Magg et al., 1972; Ringhofer and Salecker, 1980) devoted to this problem. Among these, Kroll's work plays an important role. The earlier result obtained by Dineykhan and Namsrai (1977) is based on Kroll's prescription. The essence of this procedure consists in the following.

1. To satisfy the conditions of gauge invariance for the modified theory (with the changing propagators of the charged particles) one must change the form of the one-photon vertex (for example, in the case of QED):

$$\gamma_\mu \to U_\mu(q,k) = -d_\mu(k) S_R^{-1}(\hat{q}) \tag{10}$$

(Figure 1) due to the Ward–Takahashi identity

$$k_\mu \Gamma_\mu(p,q) = S_R(\hat{p}) - S_R(\hat{q}) \tag{11}$$

where

$$\Gamma_\mu(p,q) = S_R(\hat{p}) U_\mu(k,q) S_R(\hat{q}) \qquad (p = k+q) \tag{12}$$

Here $d_\mu(k)$ is some operator whose action on the entire functions is determined below.

2. Any theory with the modified propagators and the vertex functions contains the minimal number of the many-photon vertices $e^n U_n$ satisfying the condition

$$U_n(q; k_1, \ldots, k_n) = -d(k_n) U_{n-1}(q; k_1, \ldots, k_{n-1}) \tag{13}$$

with $U_0 = S_R^{-1}$. If S_R^{-1} and U_1 are polynomial functions then the minimal

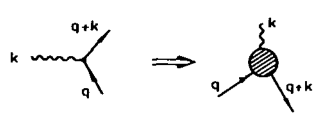

Fig. 1.

(Kroll, 1966), we obtain

$$d_\mu(k)V(-q^2l^2)=\left[V(-(q+k)^2l^2)-V(-q^2l^2)\right]\frac{\hat{k}\gamma_\mu}{k^2} \tag{15}$$

5. The d operation for the inverse of entire functions. Acting on the identity

$$V^{-1}(-q^2l^2)V(-q^2l^2)=1$$

by the d operation we get

$$d_\mu(k)V^{-1}(-q^2l^2)=-V^{-1}(-(q+k)^2l^2)\left[d_\mu(k)V(-q^2l^2)\right]V^{-1}(-q^2l^2) \tag{16}$$

6. Calculation of $d_\mu(k)S_R(\hat{q})$ where $S_R(\hat{q})$ is the modified propagator of charged particles. From the identity

$$d_\mu(k)\left[S_R(\hat{q})S_R^{-1}(\hat{q})\right]=0$$

it follows that

$$d_\mu(k)S_R(\hat{q})=-S_R(\hat{q}+\hat{k})\left[d_\mu(k)S_R^{-1}(\hat{q})\right]S_R(\hat{q}) \tag{17}$$

or

$$d_\mu(k)S_R(\hat{q})=S_R(\hat{q}+\hat{k})\Gamma_{1\mu}(k,q)S_R(\hat{q})$$

where

$$\Gamma_{1\mu}(k,q)=U_{1\mu}(k,q)=-d_\mu(k)S_R^{-1}(\hat{q})$$

It is a particular case of (14) at $n=1$.

7. The proof of validity of the generalized Ward–Takahashi identity:

$$(p_\mu-q_\mu)\Gamma_\mu(p,q)=S_R(\hat{p})-S_R(\hat{q}) \tag{18}$$

where

$$\Gamma_\mu(p,q)=S_R(\hat{p})U_\mu(k,q)S_R(\hat{q}), \qquad k=p-q$$

Taking into account the relation

$$U_\mu(k,q)=\gamma_\mu V^{-1}(-q^2l^2)+(m-\hat{q}-\hat{k})V^{-1}(-p^2l^2)$$
$$\times\left[d_\mu(k)V(-q^2l^2)\right]V^{-1}(-q^2l^2) \tag{19}$$

and equation (15) we obtain after some calculations the identity (18).

8. Charged closed loop: first of all notice that due to Kroll (1966) the closed loop in the local theory is given by

$$\Pi_n(k_1,\ldots,k_n)=\frac{1}{n}\int d^4q\,\mathrm{Sp}\{\Gamma_n(q;k_1,\ldots,k_n)S_R(\hat{q})\}$$

where

$$S_R\left(\hat{q}+\sum_{i=1}^{n}\hat{k}_i\right)\Gamma_n(q;k_1,\ldots,k_n)S_R(\hat{q})=(-1)^n d(k_1)\cdots d(k_n)S_R(\hat{q})$$

A generalization of this equation to the modified theory with entire form factors represents no difficulty and the charged loop is determined by the following expression:

$$\Pi_n^R(k_1,\ldots,k_n)=\frac{1}{n}\int d^4q\,\mathrm{Sp}\{\Gamma_n^R(q;k_1,\ldots,k_n)S_R(\hat{q})\} \tag{20}$$

where

$$\Gamma_n^R(q;k_1,\ldots,k_n)=V(-q_n^2l^2)\Gamma_n(q;k_1,\ldots,k_n)$$
$$S_R(\hat{q}_n)\Gamma_n(q;k_1,\ldots,k_n)S_R(\hat{q})$$
$$=\mathcal{P}_n\sum_{j=1}^{n}\frac{1}{j!(n-j)!}S_R(\hat{q}_n)$$
$$\times U_j(q_j;k_{j+1},\ldots,k_n)S_R(\hat{q}_j)\Gamma_{n-j}(q;k_1,\ldots,k_j)S_R(\hat{q})$$
$$=(-1)^n d(k_1)\cdots d(k_n)S_R(\hat{q})$$

with $\Gamma_0=S_R^{-1}(\hat{q})$. Tensor indexes are omitted here.

Thus we have generalized Kroll's prescription for our case and have obtained the necessary algebraical relations which provide gauge invariance for the S matrix in any order of the perturbation series. Investigations of gauge invariance of the S matrix for concrete interactions will be given below.

4. THE INTERACTION LAGRANGIAN AND CONSTRUCTION OF THE S-MATRIX

According to the above deduction, we must construct all physical quantities (for example, interaction Lagrangian, causal Green function, etc.) by means of the nonlocal fields $\varphi_R(x)$ which are associated with the fields $\varphi(\hat{x})$ by formula (3). The causal Green function of the field $\varphi_R(x)$ is determined by expression (5):

$$\mathcal{D}_R(x-y)=\langle 0|T(\varphi_R(x)\varphi_R(y))|0\rangle$$

in the physical space, i.e., in the space of a large scale, where

$$\mathcal{D}_R(x-y)=\int d^4q e^{iq(x-y)}V(-q^2l^2)\Delta(q) \tag{21}$$

is the Efimov nonlocal Green function if $V(z)=|\tilde{K}(z)|^2$ is an entire function and $\Delta(q)$ is the Fourier transform of the local Green function.

The Lagrangian of a system of fields is constructed in terms of the averaged fields $\Psi=\varphi_R(x)=\langle\varphi(\hat{x})\rangle_R$ in the Minkowski space. Thus the initial Lagrangian describing the electromagnetic and weak interactions of leptons is chosen in the form

$$\begin{gathered}
\mathcal{L}(x)=\mathcal{L}_0(x)+\mathcal{L}_{em}(x)+\mathcal{L}_w(x)\\
\mathcal{L}_0=-\frac{1}{2}:[\partial_\beta A_\alpha(x)][\partial_\beta A_\alpha(x)]:+\sum_j:\overline{\Psi}_j(x)(i\hat{\partial}-m_j)\Psi_j(x)\\
\mathcal{L}_{em}=-e:\overline{\Psi}(x)\hat{A}(x)\Psi(x):,\qquad \mathcal{L}_w=\frac{G}{\sqrt{2}}:(\overline{\Psi}(x)0_\alpha\nu(x))(\bar{\nu}(x)0_\alpha\Psi(x))
\end{gathered} \tag{22}$$

where $A_\alpha(x)$ and $\Psi_j(x)$, $\nu(x)$ are the nonlocal fields of photon and leptons. The summation in (22) runs over all considered fermion fields ($j=e,\mu,\nu_e,\nu_\mu$).

Formally, the S matrix can be written in the form of the T products:

$$\begin{gathered}
S=1+i\sum_{n=1}^{\infty}\frac{1}{n!}S_n\\
S_n=i^{n-1}\int dx_1\cdots\int dx_n T_d\left\{\prod_{j=1}^{n}[\mathcal{L}_{em}(x_j)+\mathcal{L}_w(x_j)]\right\}
\end{gathered} \tag{23}$$

Here the symbol T_d means the so-called Wick T product or T^* operation (see, for example, Bogolubov and Shirkov, 1959; Efimov, 1977) and the lower case d corresponds to the algebraic prescription determined in Section 3. If the form factor (6) is chosen as an entire function, then the proof of unitarity and causality conditions in our scheme proceeds by the same method as that of Alebastrov and Efimov (1973, 1974). Quantization of such a system has been carried out in detail by Efimov (1974).

In order to construct the perturbation series for the S matrix (23) by prescriptions of the usual local theory, it is necessary to change (in the Feynman diagrams)

$$\frac{m+\hat{k}}{m^2-k^2-i\varepsilon} \to \frac{m+\hat{k}}{m^2-k^2-i\varepsilon} V_m(-k^2l^2), \qquad \frac{g_{\mu\nu}}{-k^2-i\varepsilon} \to g_{\mu\nu}\frac{V_0(-k^2l^2)}{-k^2-i\varepsilon}$$

$$\frac{\hat{k}}{-k^2-i\varepsilon} \to \frac{\hat{k}}{-k^2-i\varepsilon} V_0(-k^2l^2)$$

and at the same time to insert the modified function (10) into the verticies at the external photon lines. The calculations of the matrix elements for the charged lepton loops will be carried out using the formulas (20) and (14).

5. INVESTIGATION OF THE PERTURBATION SERIES FOR THE S-MATRIX IN THE QUANTUM ELECTRODYNAMICS OF PARTICLES WITH SPINS 1/2, 0, AND 1

5.1. The Spinor Electrodynamics (QED). The construction of the perturbation series for the S matrix is possible only within the framework of an intermediate regularization procedure. We shall use the regularization procedure of Alebastrov and Efimov (1973). The regularizations introduced there make it possible to pass to the Euclidean metrics in any diagram of the perturbation theory. We recall that the form factors $V(-q^2l^2)$ decrease only in the Euclidean direction, i.e., when $q^2 \to -\infty$. Therefore we shall investigate the Feynman diagrams in the Euclidean momentum space.

Let us calculate the matrix elements for the S matrix corresponding to the following primitive diagrams (see Figure 3) which are divergent in the usual quantum electrodynamics.

5.2. The Diagrams of Vacuum Polarization. In the gauge-invariant stochastic theory the vacuum polarization in the second order of the perturbation theory is determined by diagrams sketched in Figure 3a. In the momentum representation the term of the S matrix which corresponds to

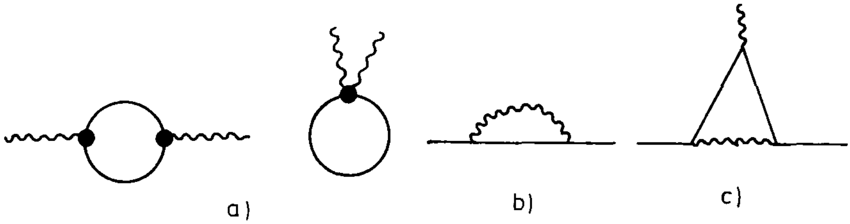

Fig. 3.

these diagrams is given by an expression of the type (20):

$$\Pi^R_{\mu\nu}(k_1,k_2)=\lim_{\delta\to 0}\frac{ie^2}{(2\pi)^4}\frac{1}{2}\int d^4q V^\delta_m(-q^2l^2)\mathrm{Sp}\left[\Gamma^\delta_{\mu\nu}(q;k_1,k_2)S^\delta_R(\hat{q})\right]$$

$$(k_1+k_2=0) \qquad (24a)$$

where

$$S(\hat{q}_2)\Gamma_{\mu\nu}(q;k_1,k_2)S(\hat{q})=(-1)^2 d_\mu(k_1)d_\nu(k_2)S(\hat{q})$$

$$V^\delta_m(-q^2l^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\zeta\frac{v(\zeta)}{\sin\pi\zeta}e^{\delta\zeta^2}\left[(m^2-q^2-i\varepsilon)l^2\right]^\zeta$$
$$(0<\beta<1)$$

$$q_2=q+k_1+k_2=q,\ S\equiv S_R \qquad (24b)$$

Equation (24a) is simplified by the d operation determined in Section 3. The regularization procedure δ guarantees the possibility of passing to the Euclidean metric. Taking thus the trace, integrating over d^4q, and going to the Euclidean metric we obtain (in the limit $\delta\to 0$, detailed calculations are given by Dineykhan and Namsrai, 1977)

$$\Pi^R_{\mu\nu}(k)=\frac{e^2}{2\pi^2}\left(k_\mu k_\nu-g_{\mu\nu}k^2\right)\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}(m^2l^2)^\zeta$$
$$\times\int_0^1 dx\, x(1-x)^{1-\zeta}\frac{\Gamma(-\zeta)}{\Gamma(1-\zeta)}\mathcal{L}_0^\zeta$$

where $\mathcal{L}_0=1-(k^2/m^2)x(1-x)$.

Assuming $m^2l^2\ll 1$ we get

$$\Pi^R_{\mu\nu}(k)=\frac{e^2}{2\pi^2}\left(k_\nu k_\mu-g_{\mu\nu}k^2\right)\int_0^1 dx\, x(1-x)$$
$$\times\left[\log\frac{\mathcal{L}_0(x)}{x(1-x)}-\frac{5}{6}+v'(0)+\log m^2l^2+O(m^2l^2)\right]$$

We see that after the charge renormalization the value obtained for the vacuum polarization coincides with the renormalized expression in the usual local theory (see, for example, Bogolubov and Shirkov, 1959).

5.3. The Diagram of Self-Energy (Figure 3b). The corresponding term in the S matrix can be written in the form

$$-i{:}\bar{\Psi}(x)\Sigma_R(x-y)\Psi(y){:}$$

where

$$\Sigma_R(x-y)=\frac{1}{(2\pi)^4}\int d^4p e^{ip(x-y)}\tilde{\Sigma}_R(p)$$

Here

$$\tilde{\Sigma}_R(p)=\lim_{\delta\to 0}\frac{-ie^2}{(2\pi)^4}\int d^4k\frac{V_0^\delta(-k^2l^2)}{-k^2-i\varepsilon}\gamma_\mu\frac{m+\hat{p}-\hat{k}}{m^2-(p-k)^2-i\varepsilon}\gamma_\mu$$
$$\times V_m^\delta(-(p-k)^2l^2)$$

We can use the representation (24b) for the form factors $V_0^\delta(-q^2l^2)$ and $V_m^\delta(\cdots)$ and the general Feynman parametric formula

$$b_1^{-\mu_1}\cdots b_n^{-\mu_n}=\frac{\Gamma(\mu_1+\cdots+\mu_n)}{\Gamma(\mu_1)\cdots\Gamma(\mu_n)}\int_0^1 d\alpha_1\cdots\int_0^1 d\alpha_n\delta\left(1-\sum_{i=1}^n\alpha_i\right)\alpha_1^{\mu_1-1}\cdots$$
$$\times\alpha_n^{\mu_n-1}\left[\sum_{j=1}^n\alpha_j b_j\right]^{-\mu_1-\cdots-\mu_n}$$

Then, after some calculations we get

$$\tilde{\Sigma}_R(p)=\frac{e^2}{8\pi^2}\frac{1}{2i}\int_{\substack{-\beta+i\infty\\(0<\beta<1)}}^{-\beta-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}(m^2l^2)^\zeta$$
$$\times\frac{1}{2i}\int_{\substack{-\gamma+i\infty\\(0<\gamma<1)}}^{-\gamma-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}(m^2l^2)^\eta\frac{\Gamma(-\eta-\zeta)}{\Gamma(1-\eta)\Gamma(1-\zeta)}$$
$$\times\int_0^1 dx\left(\frac{1-x}{x}\right)^\zeta\left(1-\frac{p^2}{m^2}x\right)^{\zeta+\eta}(2m-\hat{p}x)\qquad(25)$$

Assuming the value of m^2l^2 to be small, one can obtain for the self-energy the following expressions:

$$\tilde{\Sigma}_R(p)=\frac{e^2}{8\pi^2}\int_0^1 dx(2m-\hat{p}x)\log\frac{m^2}{m^2-p^2x}$$
$$+\frac{e^2m}{16\pi^2}\left\{\left[3\log\frac{1}{m^2l^2}-3v'(0)-1+\frac{\pi}{2i}\right.\right.$$
$$\left.\left.\times\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)v(-\zeta)}{\sin^2\pi\zeta}(3-\zeta)\right]+O(m^2l^2)\right\}$$
$$+\frac{e^2}{16\pi^2}(m-\hat{p})\left\{\left[\log\frac{1}{m^2l^2}-v'(0)+1+\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\right.\right.$$
$$\left.\left.\times\frac{v(\zeta)v(-\zeta)}{\sin^2\pi\zeta}(1-\zeta)\right]+O(m^2l^2)\right\} \tag{26}$$

The calculation of integrals of the type

$$\frac{\pi}{2i}\int_{\substack{-\beta+i\infty\\(0<\beta<1)}}^{-\beta-i\infty}d\zeta\frac{v(\zeta)v(-\zeta)}{\sin^2\pi\zeta}(\cdots)$$

is simple for a concrete choice of the function $v(\zeta)$. For example, for

$$V_b(-k^2l^2)=\left[\frac{\sin(-k^2l^2)^{1/2}}{(-k^2l^2)^{1/2}}\right]^2=\frac{1}{2i}\int_{\substack{-\gamma+i\infty\\(0<\gamma<1)}}^{-\gamma-i\infty}d\zeta\frac{v_b(\zeta)}{\sin\pi\zeta}(-k^2l^2)^{\zeta}$$

where

$$v_b(\zeta)=2^{1+2\zeta}/\Gamma(3+2\zeta) \tag{27}$$

the first integral in (26) is of the form

$$\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)v(-\zeta)}{\sin^2\pi\zeta}(3-\zeta)=-\frac{23}{9}-\sum_{n=2}^{\infty}\frac{n-3}{n(4n^2-1)(n^2-1)}$$
$$\simeq-\frac{229}{90}$$

Incidentally, we notice that the form factor V_b describes the spreaded electron as the uniformly charged ball with radius l (Efimov, 1977).

Thus the value obtained for the self-energy differs slightly from the value calculated in the Efimov's (1972) nonlocal theory.

5.4. The Vertex Diagram and the Corrections to the AMM of the Leptons and to the Lamb Shift. Let us consider the vertex diagram shown in Figure 3c. In the momentum representation it has the following form

$$\tilde{\Gamma}_{\mu}^{R}(p,q)=\frac{e^2}{(2\pi)^4 i}\int d^4k\, \mathfrak{D}\left(-(p-k)^2 l^2\right)\gamma_\nu d_\mu(q)S(\hat{k})\gamma_\nu$$

where

$$d_\mu(q)S(\hat{k})=\frac{1}{m-\hat{k}-\hat{q}}\gamma_\mu\frac{V_m(-k^2l^2)}{m-\hat{k}}$$

$$+\frac{1}{m-\hat{k}-\hat{q}}\left[V_m\left(-(k+q)^2l^2\right)-V_m(-k^2l^2)\right]\frac{\hat{q}\gamma_\mu}{q^2}$$

The intermediate regularization procedure δ is omitted here and below. By using the identity

$$a^n-b^n=n(a-b)\int_0^1 dx[(a-b)x+b]^{n-1} \tag{28}$$

the difference of the form factor values can be transformed to the form

$$V_m\left(-(k+q)^2l^2\right)-V_m(-k^2l^2)=-\left[q^2+2(k\cdot q)\right]$$

$$\times\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}l^{2\zeta}\zeta$$

$$\times\int_0^1 dx\left[m^2-k^2-2x(k\cdot q)-q^2x\right]^{\zeta-1}$$

which is convenient for calculations.

Carrying out the necessary estimates we obtained the usual form for the matrix element of the vertex functions between two free single-lepton states:

$$\tilde{\Gamma}^R_\mu(p,q)=\bar{U}_j(p')\left\{\gamma_\mu F_1(q^2)+\frac{i}{2m_j}\sigma_{\mu\nu}q^\nu F_2(q^2)\right\}U_j(p) \tag{29}$$

where $F_1(q^2)=f_1(q^2)+f_2(q^2)$, $F_2(q^2)=g_1(q^2)+g_2(q^2)$. Here

$$\sigma_{\mu\nu}=\frac{1}{2i}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$$

$$f_1(q^2)=N(\zeta,\eta)\left\{\left[-2+8\beta-2\beta^2-2\frac{q^2}{m_j^2}(1-\gamma)(1-\alpha)\right]\Gamma(1-\eta-\zeta)\right.$$
$$\left.\times\mathcal{L}_1^{-1+\eta+\zeta}-2\Gamma(-\eta-\zeta)\mathcal{L}_1^{\eta+\zeta}\right\}$$

$$g_1(q^2)=4N(\zeta,\eta)\beta(1-\beta)\Gamma(1-\eta-\zeta)\mathcal{L}_1^{-1+\eta+\zeta}$$

$$f_2(q^2)=N(\zeta,\eta)\eta\int_0^1 dt\left\{\left[2\frac{q^2}{m_j^2}-2\frac{q^2}{m_j^2}(\alpha+t\gamma)\right](1-\beta-2\alpha-2t\gamma)\right.$$
$$\left.\times\Gamma(1-\eta-\zeta)\mathcal{L}_2^{-1+\eta+\zeta}-2\Gamma(-\eta-\zeta)\mathcal{L}_2^{\eta+\zeta}\right\}$$

$$g_2(q^2)=4N(\zeta,\eta)\eta\int_0^1 dt\,\beta[1-\beta-2\alpha-2t\gamma]\Gamma(1-\eta-\zeta)\mathcal{L}_2^{-1+\eta+\zeta}$$

$$N(\zeta,\eta)=\frac{e^2}{(2\pi)^4}\frac{\pi^2}{2i}\int_{-\delta+i\infty}^{-\delta-i\infty}d\zeta\frac{v(\zeta)\left(m_j^2l^2\right)^\zeta}{\sin\pi\zeta}\frac{1}{2i}\int_{-\rho+i\infty}^{-\rho-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}\left(m_j^2l^2\right)^\eta$$
$$\times\frac{1}{\Gamma(1-\zeta)\Gamma(1-\eta)}\iiint_0^1 d\alpha\,d\beta\,d\gamma\,\beta^{-\zeta}\gamma^{-\eta}\delta(1-\alpha-\beta-\gamma)$$

$$\mathcal{L}_2(q^2)=\mathcal{L}_1(q^2)-\frac{q^2}{m_j^2}t\gamma(1-\gamma t)+2\frac{q^2}{m_j^2}\alpha t\gamma+\frac{q^2}{m_j^2}t\gamma\beta$$

$$\mathcal{L}_1(q^2)=(1-\beta)^2-\frac{q^2}{m_j^2}\alpha\gamma,\qquad (m_j=m_e,m_\mu)$$

The first term of (29) in the limit $q^2 \to 0$ and with the assumption $m_j^2 l^2 \ll 1$ is

$$F_1(q_2)$$

$$= -\frac{\alpha}{4\pi}\left\{\log\frac{1}{m_j^2 l^2} - 2\log\frac{m_j^2}{m_\gamma^2} - v'(0) + \frac{9}{4}\right.$$

$$\left. - m_j^2 l^2 v(1)\left[-\frac{4}{3}v'(0) + 4\frac{v'(1)}{v(1)} - \frac{8}{3}\log\frac{1}{m_j^2 l^2} - \frac{20}{9}\right]\right\} - \frac{\alpha}{2\pi}\frac{q^2}{m_j^2}$$

$$\times\left\{\frac{2}{3}\left(\log\frac{m_j}{m_\gamma} - \frac{5}{8}\right) + v(1)m_j^2 l^2\left[\frac{1}{3}v'(0) + \frac{2}{3}\frac{v'(1)}{v(1)} + \log m_j^2 l^2 - \frac{3}{4}\right]\right\}$$

$$\alpha = \frac{e^2}{4\pi}$$

(m_γ is photon mass) and contains the terms corresponding to the charge renormalization of the leptons. The second term of (29) at $q^2 = 0$ contributes to the AMM of the leptons by

$$a_j = F_2(0) = \frac{4}{(2i)^2}\frac{\alpha}{2\pi}\int_{-\beta+i\infty}^{-\beta-i\infty} d\zeta \frac{v(\zeta)}{\sin\pi\zeta}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\eta \frac{v(\eta)}{\sin\pi\eta}$$

$$\times\left(m_j^2 l^2\right)^{\eta+\zeta}\frac{\Gamma(1-\eta-\zeta)\Gamma(1+2\zeta+\eta)}{\Gamma(3-\eta)\Gamma(3+\eta+\zeta)}(1-\eta)(1-\zeta)$$

Assuming $m_j^2 l^2 \ll 1$ ($m_j = m_e, m_\mu$) we get

$$a_j = \frac{\alpha}{2\pi}\left\{1 + m_j^2 l^2\left[-\frac{2}{3}v(1) - \frac{2}{3}\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\zeta \frac{v(\zeta)v(1-\zeta)}{\sin^2\pi\zeta}\zeta(1-\zeta)\right]\right\}$$

$$\simeq \frac{\alpha}{2\pi}\left[1 - \frac{2}{3}m_j^2 l^2 v(1)\right] \tag{30}$$

The present experimental values of lepton AMM (Van Dyck et al., 1977; Bailey et al., 1978)

$$a_{\exp}(e^-) = (1{,}159{,}652.41 \pm 0.2)10^{-9}, \qquad a_{\exp}(\mu) = (1{,}165{,}924 \pm 8.5)10^{-9}$$

are reliably confirmed by quantum electrodynamics (Calmet et al., 1977;

Kinoshita, 1979). It is natural to suppose that the contributions calculated here should be of an order not greater than the experimental errors. This makes it possible to establish the following restrictions on the parameter l:

$$l \lesssim \begin{cases} 4.9\times 10^{-14}\ \text{cm} & \text{for } V=V_1 \\ 4.5\times 10^{-14}\ \text{cm} & \text{for } V=V_s \end{cases} (e^-)$$

$$l \lesssim \begin{cases} 1.5\times 10^{-15}\ \text{cm} & \text{for } V=V_1 \\ 1.4\times 10^{-15}\ \text{cm} & \text{for } V=V_s \end{cases} (\mu)$$

Here $V=V_1$ is given by (see Namsrai, 1981a)

$$\begin{aligned} V_1 &= \sin^4\left[l(m^2-q^2)^{1/2}\right] \Big/ \left[l(m^2-q^2)^{1/2}\right]^4 \\ &= \frac{1}{2i}\int_{\substack{-\beta+i\infty \\ (0<\beta<1)}}^{-\beta-i\infty} d\zeta \frac{v_1(\zeta)}{\sin\pi\zeta} l^{2\zeta}(m^2-q^2-i\varepsilon)^{\zeta} \end{aligned} \tag{31}$$

$$v_1(\zeta)=\frac{2^{1+2\zeta}(2^{2+2\zeta}-1)}{\Gamma(5+2\zeta)}$$

and $V=V_s$ equals (8).

Similarly, for the level $n=2$ of the hydrogen atom the calculated shift $2S_{1/2}-2P_{1/2}$ due to (29) is (see Efimov, 1977, for detail)

$$\begin{aligned} \Delta E_l(2S_{1/2}-2P_{1/2}) &= \alpha^2 \text{Ry}\left\{m^2F_1'(0)-\frac{1}{2}F_2(0)\right\} \\ &= -\frac{\alpha^3}{2\pi}\text{Ry}\cdot m_e^2 l^2 v(1)\left[\frac{1}{3}v'(0)+\frac{2}{3}\frac{v'(1)}{v(1)}+\log m_e^2 l^2-\frac{13}{12}\right] \end{aligned}$$

For the function $v_s(\zeta)$ determined by (9) this expression acquires the form

$$\Delta E_l(2S_{1/2}-2P_{1/2})=\frac{\alpha^3}{2\pi}\text{Ry}\cdot\frac{m_e^2l^2}{5}\left(\log\frac{1}{m_e^2l^2}+4.15\right) \tag{32}$$

where

$$\alpha^3\text{Ry}=m_e\alpha^5/2=1.25\times 10^3\ \text{MHz}$$

The observed shift of 1057.912±0.011 MHz is well explained by QED

(Brodsky and Drell, 1970; see also Scadron, 1980). Therefore $|\Delta E_l(2S_{1/2} - 2P_{1/2})| \lesssim 0.011$ MHz and substituting the formula (32) into this inequality we get $l \lesssim 1.9 \times 10^{-13}$ cm.

The S matrix obtained is gauge invariant. Indeed, in the stochastic electrodynamics under consideration the Ward identity

$$\frac{\partial}{\partial p_\mu} \tilde{\Sigma}_R(p) = -\tilde{\Gamma}_\mu^R(p,0)$$

is valid because this identity is a direct consequence of the identities (17) and (10). Since we must not do any subtractions of infinite counterterms, no dangerous terms which can break the Ward identity when the formula (17) is valid will appear in the perturbation theory. The diagram of the vacuum polarization is gauge invariant due to our choice of the gauge-invariant regularization procedure of Kroll.

6. THE ELECTRODYNAMICS OF PARTICLES WITH SPINS 0 and 1

Now let us construct within the framework of our approach the gauge-invariant quantum electrodynamics of particles with spins 0 and 1 on the basis of the first-order Duffin–Kemmer equations (see Efimov and Namsrai, 1975).

Investigation of the perturbation theory for the S matrix constructed on the basis of Duffin–Kemmer equations will be formally the same as in QED of leptons. Therefore, in this case we should use the methods and procedures developed in constructing the gauge-invariant spinor electrodynamics in terms of stochastic space concept.

Hence, as was shown above, the averaged field $\Psi_R(x)$ of a boson field in the space $R_4(\hat{x})$ leads to the change of the free-particle propagator (in momentum representation):

$$T(\hat{p}) = \frac{\hat{p}(\hat{p}+m) - p^2 + m^2}{m(m^2 - p^2 - i\varepsilon)} \to V_m(-p^2 l^2) T(\hat{p}) \equiv T_R(\hat{p})$$

where $\hat{p} = p_\mu \beta_\mu$ and β_μ are the four 16-rank matrices which are split into five- and ten-rank matrices for particles with spins 0 and 1, respectively. In this case it is also necessary to change the form of the one-photon vertex:

$$\beta_\mu \to U_\mu(q,k) = -d_\mu(k) T_R^{-1}(\hat{q})$$

Here the following identities hold:

$$d_\mu(k)T_R(\hat{q})=T_R(\hat{q}+\hat{k})U_\mu(q,k)T_R(\hat{q})$$
$$(p_\mu-q_\mu)\Gamma_\mu^R(p,q)=T_R(\hat{p})-T_R(\hat{q}) \tag{33}$$

where

$$\Gamma_\mu^R(p,q)=T_R(\hat{p})U_\mu(k,q)T_R(\hat{q}),\qquad k=p-q$$
$$U_\mu(k,q)=\beta_\mu V^{-1}(-q^2l^2)+(m-\hat{q}-\hat{k})V^{-1}(-p^2l^2)$$
$$\times\left[d_\mu(k)V(-q^2l^2)\right]V^{-1}(-q^2l^2)$$

and

$$d_\mu(k)V(-q^2l^2)=\{V(-p^2l^2)-V(-q^2l^2)\}\frac{k_\mu-2\beta_\mu\hat{k}+2\hat{k}\beta_\mu}{k^2} \tag{34}$$

Let us now examine the perturbation series for the S matrix.

6.1. The Diagrams of the Vacuum Polarization for Boson Fields. The expression (24) for boson fields becomes

$$\tilde{\Pi}_{\mu\nu}^R(k_1,k_2)=\lim_{\delta\to 0}\frac{ie^2}{(2\pi)^4}\int d^4qV_m^\delta(-q_2^2l^2)\mathrm{Sp}\left[\Gamma_{\mu\nu}^\delta(q,k_1,k_2)T_R^\delta(\hat{q})\right] \tag{35}$$

where $k_1+k_2=0$, $q_2=q+k_1+k_2=q$,

$$T_R(\hat{q}_2)\Gamma_{\mu\nu}(q,k_1,k_2)T_R(\hat{q})=(-1)^2d_\mu(k_1)d_\nu(k_2)T_R(\hat{q})$$

Making use of the definition of the d_μ operation (34) for an entire functions and

$$\mathrm{Sp}\{\beta_\mu\beta_\nu\beta_\lambda\beta_\sigma\cdots\beta_\rho\beta_\tau\beta_\chi\}=\begin{cases}g_{\mu\nu}g_{\lambda\sigma}\cdots g_{\tau\chi}+g_{\nu\lambda}g_{\sigma\ldots}\cdots g_{\rho\tau}g_{\chi\mu} & \text{if } n \text{ is even}\\ 0 & \text{if } n \text{ is odd}\end{cases}$$

for the scalar boson,

$$\mathrm{Sp}\,\beta_\nu\beta_\mu = 6g_{\mu\nu}$$
$$\mathrm{Sp}\,\beta_\nu\beta_\mu\beta_\lambda\beta_\rho = 3(g_{\mu\nu}g_{\lambda\rho} + g_{\mu\lambda}g_{\nu\rho})$$
$$\vdots$$

for the vector boson, and performing necessary calculations we get

$$\tilde{\Pi}^{(s)}_{\mu\nu}(k) = -\frac{\alpha}{4\pi}\left(g_{\mu\nu}k^2 - k_\mu k_\nu\right)\frac{1}{2i}\int_{\substack{-\gamma+i\infty\\(0<\gamma<1)}}^{-\gamma-i\infty} d\zeta\,\frac{v(\zeta)(m^2l^2)^\zeta}{\sin\pi\zeta} \times \frac{\Gamma(-\zeta)}{\Gamma(1-\zeta)}\int_0^1 dx(1-x)^{-\zeta}(1-2x)^2\mathfrak{L}_0^\zeta$$

for the scalar boson, and

$$\tilde{\Pi}^{(v)}_{\mu\nu}(k) = -\frac{\alpha}{4\pi}\left(g_{\mu\nu}k^2 - k_\mu k_\nu\right)\frac{1}{2i}\int_{\substack{-\gamma+i\infty\\(1<\gamma<2)}}^{-\gamma-i\infty} d\zeta\,\frac{v(\zeta)}{\sin\pi\zeta}(m^2l^2)^\zeta \times \int_0^1 dx(1-x)^{-\zeta}\{3(1-2x)^2 \times \Gamma(-\zeta)/\Gamma(1-\zeta) + 2[\Gamma(-1-\zeta)/\Gamma(1-\zeta)]\mathfrak{L}_0\}\mathfrak{L}_0^\zeta$$

for the vector boson and $\mathfrak{L}_0 = 1 - k^2x(1-x)/m^2$ (see Dineykhan et al., 1977, for detail).

Assuming the value of $m_j^2l^2$ ($m_j = m_s, m_v$) to be small we obtain finally

$$\tilde{\Pi}^{(s)}_{\mu\nu}(k) = \frac{\alpha}{24\pi}\left(k^2g_{\mu\nu} - k_\mu k_\nu\right) \times \left\{\frac{k^2}{m^2}\int_0^1 dy\,\frac{y^4}{1-(k^2/m^2)(1-y^2)} + 2\left[v'(0) + \log m^2l^2\right] + \frac{3}{8}\right\}$$

$$\tilde{\Pi}^{(v)}_{\mu\nu}(k) = \frac{\alpha}{4\pi}\left(k^2g_{\mu\nu} - k_\mu k_\nu\right) \times \int_0^1 dx\left\{\left(2 - 2\frac{k^2}{m^2}x(1-x) - 3(1-2x)^2\right)\log\left[1 - \frac{k^2}{m^2}x(1-x)\right] + 2\left(2^{-1} - \frac{k^2}{6m^2}\right)\left[v'(0) + \log m^2l^2\right] - \frac{\delta}{m^2l^2} - \frac{1}{9}\frac{k^2}{m^2} + \frac{7}{3} + \frac{k^2}{m^2}x(1-x)\right\}$$

$$\delta = v'(-1)$$

It is necessary to perform the charge renormalization of the scalar boson and the charge and mass renormalization for the vector boson, and after these procedures we obtain the usual quantum-electrodynamical quantities for the vacuum polarization of these particles:

$$\tilde{\Pi}^{(s)}_{\mu\nu}(k)=\frac{\alpha}{24\pi}\frac{k^2}{m^2}\left(k^2 g_{\mu\nu}-k_\mu k_\nu\right)\int_0^1 dy\, y^4\Big/\left[1-\frac{k^2}{m^2}(1-y^2)\right]$$

$$\tilde{\Pi}^{(v)}_{\mu\nu}(k)=\frac{\alpha}{4\pi}\left(k^2 g_{\mu\nu}-k_\mu k_\nu\right)\int_0^1 dx$$

$$\times\left\{\left[2-2\frac{k^2}{m^2}x(1-x)-3(1-2x)^2\right]\log\left[1-\frac{k^2}{m^2}x(1-x)\right]\right.$$

$$\left.-3(1-2x)^2\frac{k^2}{m^2}x(1-x)+2\frac{k^2}{m^2}x(1-x)\right\}$$

6.2. The Self-Energy of the Boson. Let us now consider the self-energy diagram, shown in Figure 3b (here the solid line corresponds to the boson field). In our scheme the term of the S matrix corresponding to this diagram has the form

$$-i{:}\overline{\Psi}(x)\Sigma^b_R(x-y)\Psi(y){:}$$

$$\Sigma^b_R(x-y)=\frac{1}{(2\pi)^4}\int d^4p\, e^{ip(x-y)}\tilde{\Sigma}^b_R(p)$$

where

$$\tilde{\Sigma}^b_R(p)=\lim_{\delta\to 0}\frac{-ie^2}{(2\pi)^4}\int d^4k\frac{V^\delta_0(-k^2l^2)}{-k^2-i\varepsilon}$$

$$\times\beta_\mu\frac{m^2-(p-k)^2+(\hat{p}-\hat{k})(\hat{p}-\hat{k}+m)}{m\left[m^2-(p-k)^2-i\varepsilon\right]}\beta_\mu V^\delta_m\left(-(p-k)^2l^2\right) \tag{36}$$

Passing to the Euclidean metric in (36) and substituting representation (24b) into it in a way similar to that used in the previous section it is easy to obtain the following expression:

$$\tilde{\Sigma}_R^b(p)=\frac{\alpha m}{4\pi}\left\{\left[B+\frac{\hat{p}}{m}+\frac{\hat{p}^2}{m^2}(1-B)\right]\mathfrak{T}_{0,0}-\left[2(1-B)\frac{\hat{p}^2}{m^2}+\frac{\hat{p}}{m}\right]\mathfrak{T}_{1,0}\right.$$
$$\left.-\frac{1}{2}B(1-B)\mathfrak{T}_{1,1}+(1-B)\frac{\hat{p}^2}{m^2}\mathfrak{T}_{2,0}\right\}$$

where

$$B=\beta_\mu\beta_\mu,\qquad \hat{p}=\beta_\mu p_\mu=\beta_0 p_0-\boldsymbol{\beta}\mathbf{p}$$

$$\mathfrak{T}_{a,b}=\mathfrak{T}_{a,b}\left(m^2l^2,\frac{p^2}{m^2}\right)=\frac{1}{2i}\int_{-\beta_{a+b}+i\infty}^{-\beta_{a+b}-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}(m^2l^2)^\zeta$$
$$\times\frac{1}{2i}\int_{-\gamma_{a+b}+i\infty}^{-\gamma_{a+b}-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}(m^2l^2)^\eta H_{a,b}\left(\zeta,\eta,\frac{p^2}{m^2}\right),$$

$$H_{a,b}\left(\zeta,\eta,\frac{p^2}{m^2}\right)=\frac{\Gamma(-b-\zeta-\eta)}{\Gamma(1-\zeta)}\int_0^1 du\,u^{-\zeta}(1-u)^{\zeta+a}\left(1-\frac{p^2}{m^2}u\right)^{\zeta+\eta+b}$$

Here

$$\frac{a+b}{2}<\beta_{a+b}+\gamma_{a+b}<\frac{a+b+1}{2},\qquad a,b=0,1,2$$

We calculate the electromagnetic correction to the boson mass in the two limits $m^2l^2\ll1$ and $m^2l^2\gg1$. We have (i) for $m^2l^2\ll1$,

$$\tilde{\Sigma}_R^b(m)=\frac{3\alpha}{48\pi}B(B-1)\frac{m}{m^2l^2}\delta=\begin{cases}\dfrac{3\alpha}{4\pi}\dfrac{m}{m^2l^2}\delta & \text{for } s=0\\ -\dfrac{\alpha}{4\pi}\dfrac{m}{m^2l^2}\delta & \text{for } s=1\end{cases}$$

and (ii) for $m^2l^2\gg1$, i.e., the classical limit,

$$\tilde{\Sigma}_R^b(m)=\frac{\alpha m}{2ml}\frac{\pi}{2i}\int_{-\gamma+i\infty\ (1/2<\gamma<1)}^{-\gamma-i\infty}d\eta\frac{v(\eta)v(-1/2-\eta)}{\sin^2\pi\eta}\left[1+O\left(\frac{1}{m^2l^2}\right)\right]$$

for $s=0,1$

Here

$$\delta = \frac{\pi}{2i} \int_{\substack{-\beta+i\infty \\ (0<\beta<1)}}^{-\beta-i\infty} d\eta \frac{v(\eta)v(-1-\eta)}{\sin^2\pi\eta} \Gamma(1-\eta)\left[1+O(\log m^2 l^2)\right]$$

The vertex diagram and the statistical characteristics for the bosons were investigated by Dineykhan et al. (1977) within the framework of the stochastic space and by Efimov and Namsrai (1975) in the nonlocal theory. Here we shall not write out these results because they are quite cumbersome.

It is verified easily that in the case of the electrodynamics of particles with spins 0 and 1 gauge invariance holds automatically by construction.

7. THE CONSTRUCTION OF GAUGE-INVARIANT FOUR-FERMION THEORY OF WEAK INTERACTIONS IN THE STOCHASTIC SPACE

7.1. Introduction. The four-fermion theory (Fermi theory) plays, and it seems, will play a fundamental role in the development of the theory of weak interactions. The four-fermion V-A interaction describes in a unified way some weak decays of leptons and fermions. Earlier success of this theory in the explanation of muon and β decays has given a certain hope that within the framework of this theory weak processes might be described at least in the low-energy domain.

However, in four-fermion theory one meets the well-known difficulty caused by the ultraviolet divergences and the renormalization problem. For this reason, calculations of higher-order corrections in the perturbation series in coupling constant G are difficult. Notice that a similar situation occurs in the theory with intermediate vector bosons.

Ways of eliminating these difficulties were proposed in different models which can be classified in two groups. One of them is connected with some schemes and approaches aimed at constructing a new theory of weak interactions within the framework of gauge theory [especially, the unified theory of weak and electromagnetic interactions, i.e., the standard model of electroweak interactions due to Weinberg (1967) and Salam (1968) (see Glashow, 1980).] The other approach assumes a modification of the usual theory of weak interactions based on the analysis of fundamental principles (causality condition, locality and properties of geometry on small scale, etc.) of modern local quantum field theory at small distances (see, for example, Alebastrov et al., 1973; Kadyshevsky, 1980; Efimov and Seltser, 1971; Arbuzov, 1975).

The description of weak, electromagnetic, and also strong interactions within the quark model framework based on gauge theories with a spontaneously broken symmetry obviously represents a qualitatively new step in understanding elementary particle phenomenon and their internal structure.

However, at the present time it is impossible to say that the first approach is generally accepted and that the second way in the development of elementary particle theory loses its significance.

Maybe that usefulness of the old (free of the above-mentioned difficulties) and of new weak interaction theories is revealed mainly in two limiting cases. Indeed, when the energy is not high enough for the production of intermediate particles (for example, $W^{\pm}$, Z^0, Higgs bosons, etc.) which are necessary in gauge theories, i.e., if the energy is small with respect to some limiting value E_l, the weak processes must be described by the four-fermion theory. Contrarywise, when $E \gtrsim E_l$ the gauge theory will play an important role in weak processes. Here E_l is the value of energy at which new particles $W^{\pm}$, Z^0, etc. will be produced, if they exist. Of course, there has to exist a reasonable correspondence of both the theories at $E \sim E_l$.

In the language of distance it means that starting from some small scale $l_k \sim 1/E_l$ the growth of weak-processes cross section must be compensated by corrections given by the intermediate bosons. It is very interesting to show, at least approximately, the energy value E_l (or the distance l_k).

It seems that in view of this aim, the investigation of four-fermion theory within the framework of the second approach is undoubtly interesting and can give a new information about weak interactions. For example, it is quite possible that on the basis of this type investigations a value of l_k (or E_l) may be obtained. Moreover, recently great attention has been paid to the low-energy weak interactions. This is connected with the problem of neutrino oscillations and its consequences, and of the proton instability in the grand unified theories (see, for example, Fiorini, 1979).

This section is devoted to construction of a gauge-invariant theory of weak and electromagnetic interactions of leptons in the stochastic space $R_4(\hat{x})$ and to calculations of "weak" corrections to the AMM of leptons and to the Lamb shift within the framework of this theory. In this case the investigation of the terms of the S matrix is carried out by the methods elaborated in previous sections.

Notice that in the stochastic (nonlocal) theory the concept of stochasticity (locality breaking) is characterized not only by the length $l_k \sim 1/E_l$, but also by a form of the distributions $w(b_E^2/l^2)$ (2) (the shape of a form factor or of potential) at small distances. It seems to us that in real physical processes apart from the value of elementary length an important role may be played by the form factors of the theory. This occurred especially in the study of the decay $K_L^0 \to \mu^+\mu^-$ and of the mass difference $\Delta m(K_L^0 - K_S^0)$ in our scheme.

In the last cases our analysis of experimental data on weak processes within the discussed model shows that the elementary length of weak interactions is of an order of $l_k \sim 10^{-16}$ cm. and that the unitary limit is reached at energies $E_l \sim 100$–200 GeV depending on the choice of the form factor. Notice that it is quite possible that at these energies the unification of electromagnetic and weak interactions, which are described by the standard model of Weinberg and Salam, is achieved.

7.2. Gauge Invariance for the S Matrix in the Nonlocal Theory of Weak Interactions. The expansion of the S matrix in powers of the normally ordered operators of the electromagnetic field $A_\mu(x)$ and the lepton fields $\Psi(x)$ has the form

$$S=\sum_{n,m,l}\frac{1}{n!m!l!}\int d^4k_1\cdots\int d^4k_n\int d^4p_1\cdots\int d^4p_m\int d^4q_1\cdots$$
$$\times\int d^4q_l F_{\mu_1,\ldots,\mu_n}(k_1,\ldots,k_n;p_1,\ldots,p_m;q_1,\ldots,q_l):A_{\mu_1}(k_1)\cdots A_{\mu_n}(k_n)$$
$$\times\Psi_j(p_1)\cdots\Psi_j(p_m)\overline{\Psi}_j(q_1)\cdots\overline{\Psi}_j(q_l):\quad (j=e,\mu,\nu_e,\nu_\mu)\tag{37}$$

The requirement of gauge invariance means that the coefficient functions $F_{\mu_1,\ldots,\mu_n}(\cdots)$ in the expansion (37) satisfy the following conditions:

$$k_{\mu_j}F_{\mu_1,\ldots,\mu_j,\ldots,\mu_n}(\cdots)=0$$
$$k_{\mu_i}k_{\mu_j}F_{\mu_1,\ldots,\mu_i,\ldots,\mu_j,\ldots,\mu_n}(\cdots)=0\tag{38}$$

Let us remark that each of the conditions (38) is fulfilled when other momenta in the function $F_{\mu_1,\ldots,\mu_n}(\cdots)$ are on the mass shell. The series of the perturbation theory contains three types of diagrams: diagrams with purely weak vertices, with weak and electromagnetic, and with purely electromagnetic vertices. Investigation of the last type of diagrams will not be carried out here because they represent gauge-invariant quantum electrodynamics constructed by us in Section 5. Proof of the fulfilment of condition (38) for the diagrams with mixed weak and electromagnetic vertices goes simply. Indeed, in the considered four-fermion theory of weak interactions the Ward identity

$$\frac{\partial\tilde{\Sigma}_R(p)}{\partial p_\mu}=-\tilde{\Gamma}^R_\mu(p,0)$$

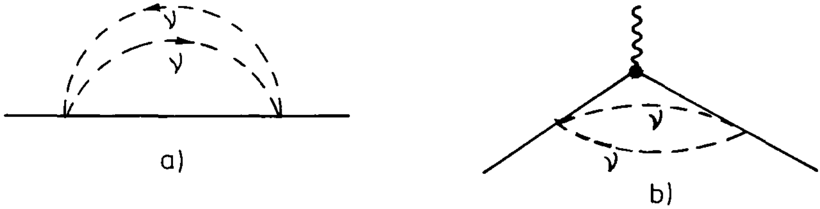

Fig. 4.

is valid since it is a consequence of the identities (17) and (10) at $k=0$: By definition:

$$d_\mu(k)|_{k\to 0}F(q)=\frac{\partial}{\partial q_\mu}F(q)$$

Here $\tilde{\Sigma}_R(p)$ and $\tilde{\Gamma}_\mu^R(p,q)$ correspond to the diagrams of the self-energy (Figure 4a) and the vertex (Figure 4b), respectively.

Proof of the gauge invariance in the form (38) in the series of perturbation theory is quite simple and is based on the identity (11). The diagrams of closed loops constructed by propagators of charged leptons are gauge invariant due to the d operation.

Now let us proceed to investigate these diagrams (see, for example, the diagrams shown in Figure 5) in the series of perturbation theory. First consider the diagram represented in Figure 5a. In the momentum representation the term of the S matrix corresponding to this diagram is given by the following expression

$$F_{\mu\alpha}^{(2)}(k)=\frac{ieG}{\sqrt{2}\,(2\pi)^4}\int d^4p\,\mathrm{Sp}\left\{f_{\mu\alpha}^{(2)}(p,k)\right\} \tag{39}$$

where

$$f_{\mu\alpha}^{(2)}=d_\mu(k)S_R(\hat{p})O_\alpha$$

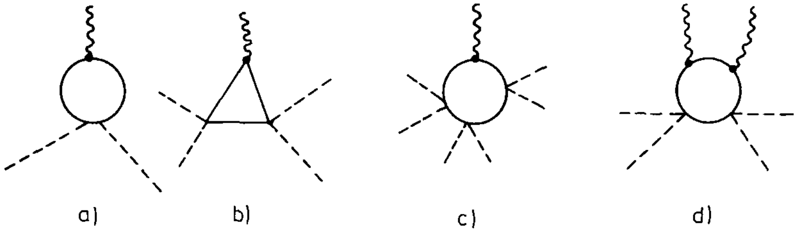

Fig. 5.

Taking into account the formulas (11) and (17) we get

$$k_\mu d_\mu(k) S_R(\hat{p}) = S_R(\hat{p}+\hat{k}) - S_R(\hat{p}) \tag{40}$$

From this it is easily seen that

$$k_\mu F^{(2)}_{\mu\alpha}(k) = 0$$

By using the main property (40) for the d operation we can easily prove the fulfillment of the gauge invariance condition for the other matrix elements of closed charged loops, shown in Figure 5. The matrix element corresponding to Figure 5b is given by

$$f^{(3)}_{\mu\alpha\beta}(p,k) = d_\mu(k)\left[S_R(\hat{p}+\hat{q})O_\alpha S_R(\hat{q})O_\beta\right] \tag{41}$$

where p, k are the external momenta, q is the internal momentum over which one carries out integration. Making use of the identity (40) we obtain

$$\begin{aligned} k_\mu f^{(3)}_{\mu\alpha\beta} &= \left[S_R(\hat{p}+\hat{q}+\hat{k}) - S_R(\hat{p}+\hat{q})\right]O_\alpha S_R(\hat{q})O_\beta + S_R(\hat{p}+\hat{k}+\hat{q})O_\alpha \\ &\quad\times\left[S_R(\hat{q}+\hat{k}) - S_R(\hat{q})\right]O_\beta = \\ &\quad - S_R(\hat{p}+\hat{q})O_\alpha S_R(\hat{q})O_\beta + S_R(\hat{p}+\hat{q}+\hat{k})O_\alpha S_R(\hat{q}+\hat{k})O_\beta \end{aligned} \tag{42}$$

Elementary integration over q gives the following identity:

$$k_\mu F^{(3)}_{\mu\alpha\beta}(p,k) = 0$$

Now we consider the fourth-order diagrams (Figure 5c). In this case the terms of types $f^{(2)}_{\mu\alpha}$ and $f^{(3)}_{\mu\alpha\beta}$ have the form

$$f^{(4)}_{\mu\alpha\beta\gamma}(k,q_1,q_2) = d_\mu(k)\left[S_R(\hat{q})O_\alpha S_R(\hat{q}+\hat{q}_1)O_\beta S_R(\hat{q}+\hat{q}_1-\hat{q}_2)O_\gamma\right]$$

From this we obtain easily

$$\begin{aligned} k_\mu f^{(4)}_{\mu\alpha\beta\gamma} &= \left[S_R(\hat{q}+\hat{k}) - S_R(\hat{q})\right]O_\alpha S_R(\hat{q}+\hat{q}_1)O_\beta S_R(\hat{q}+\hat{q}_1-\hat{q}_2)O_\gamma \\ &\quad + S_R(\hat{q}+\hat{k})O_\alpha\left[S_R(\hat{q}+\hat{q}_1+\hat{k}) - S_R(\hat{q}+\hat{q}_1)\right]O_\beta S_R(\hat{q}+\hat{q}_1-\hat{q}_2)O_\gamma \\ &\quad + S_R(\hat{q}+\hat{k})O_\alpha S_R(\hat{q}+\hat{q}_1+\hat{k})O_\beta \\ &\quad \times\left[S_R(\hat{q}+\hat{q}_1-\hat{q}_2+\hat{k}) - S_R(\hat{q}+\hat{q}_1-\hat{q}_2)\right]O_\gamma \\ &= - S_R(\hat{q})O_\alpha S_R(\hat{q}+\hat{q}_1)O_\beta S_R(\hat{q}+\hat{q}_1-\hat{q}_2)O_\gamma \\ &\quad + S_R(\hat{q}+\hat{k})O_\alpha S_R(\hat{q}+\hat{q}_1+\hat{k})O_\beta S_R(\hat{q}+\hat{q}_1-\hat{q}_2+\hat{k})O_\gamma \end{aligned}$$

In the second term of this expression, the substitution $q+k \to q$ and integration over d^4q give

$$k_\mu F^{(4)}_{\mu\alpha\beta\gamma} = 0$$

Here finally let us consider one more diagram, shown in Figure 5d. The term $f^{(2,2)}_{\mu\nu\alpha\beta}(k_1, k_2, p)$ acquires the form

$$f^{(2,2)}_{\mu\nu\alpha\beta} = d_\nu(k_2)\left\{d_\mu(k_1)\left[O_\alpha S_R(\hat{p}+\hat{q})O_\beta S_R(\hat{q})\right]\right\} \tag{43}$$

From this we see that due to equality (42) we obtain the following identity:

$$k_{1\mu}k_{2\nu}F^{(2,2)}_{\mu\nu\alpha\beta} = 0$$

Thus, the algebraic relations we have found show that within our model the terms of the S matrix satisfy the condition of gauge invariance in the form (38) in each perturbation order.

7.3. Calculation of the "Weak" Corrections to the AMM of the Leptons and to the Lamb Shift. In view of testing locality of the quantum theory, calculation of corrections due to the weak interactions to the quantum electrodynamical processes is always very interesting. In the case of an actual discrepancy between the local quantum electrodynamics and an experiment, which is to be expected at very high energies, the "weak" corrections would contain, however, just the breakdown of QED. From this the conclusion may be drawn that local QED can be violated, and weak and electromagnetic forces can be equal in magnitude. It is quite possible that in this domain of energies the process of unification of weak and electromagnetic interactions starts.

The present section is devoted to the calculation of corrections to the AMM of the leptons, and to the Lamb shift, and further to establishment of the lower bound for the parameter l which characterizes a domain of unification of weak and electromagnetic interactions.

7.3.1. The AMM of Leptons. In the lowest order of G the corrections to the anomaly of leptons are given by two types of diagrams (Figure 6a) corresponding to both the diagonal and nondiagonal weak interactions, i.e., to two types of terms of the interaction Lagrangian $\mathfrak{L}_w$ in (22). Within the framework of the nonlocal theory the diagrams of such a type were discussed in detail by Efimov et al. (1973). Therefore we shall not calculate in detail, and give only the main result. Hence in the stochastic theory the

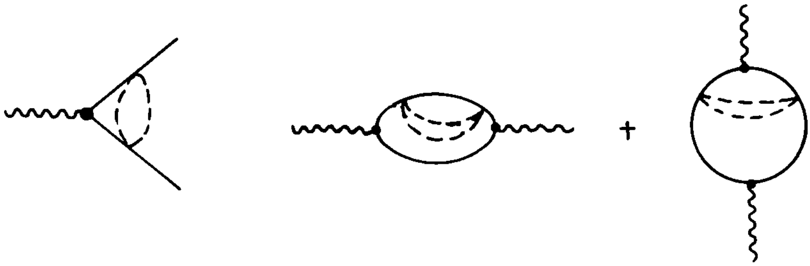

Fig. 6.

terms corresponding to these diagrams may be rewritten in the form

$$\tilde{\Gamma}_{\mu}^{(j)}(p,q)=\lim_{\delta\to 0}\frac{G^2}{2i(2\pi)^4}\int d^4kN_{\alpha\beta}^{\delta}(k)O_{\alpha}S^{\delta}(\hat{p}'+\hat{k})U_{\mu}^{\delta}(p,k)S^{\delta}(\hat{p}+\hat{k})O_{\beta}$$

$$S\equiv S_R,\qquad j=e,\mu \tag{44}$$

where the symbol δ means the intermediate regularization procedure used above. Here p and p' are the external momenta of the leptons, $p'=p+q$ and

$$S(\hat{p}'+\hat{k})U_{\mu}(p,k)S(\hat{p}+\hat{k})=-S(\hat{p}'+\hat{k})\frac{\gamma_{\mu}}{m_j-\hat{p}-\hat{k}}$$

$$+\left[V_{m_j}\left(-(p'+k)^2l^2\right)-V_{m_j}\left(-(p+k)^2l^2\right)\right]\frac{\hat{q}\gamma_{\mu}+2p_{\mu}}{q^2+2(p\cdot q)}\frac{1}{m_j-\hat{p}-\hat{k}} \tag{45}$$

The function $N_{\alpha\beta}(k)$ corresponds to the neutrino loop and is given by (see below Section 8.2)

$$N_{\alpha\beta}(k)=\frac{1}{2\pi^2}\frac{1}{2i}\int_{\substack{-\alpha+i\infty\\(0<\alpha<1)}}^{-\alpha-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}(-l^2k^2)^{\zeta}\frac{1}{2i}\int_{\substack{-\beta+i\infty\\(0<\beta<1)}}^{-\beta-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}$$

$$\times(-l^2k^2)^{\eta}\frac{\Gamma(2+\zeta)\Gamma(2+\eta)}{\Gamma(1-\zeta)\Gamma(1-\eta)}\frac{1}{\Gamma(4+\zeta+\eta)}$$

$$\times\left[-2k_{\alpha}k_{\beta}\Gamma(-\eta-\zeta)-g_{\alpha\beta}k^2(2+\eta+\zeta)\Gamma(-1-\eta-\zeta)\right] \tag{46a}$$

Here $\mathrm{Re}(-\eta-\zeta)=\alpha+\beta<0$ and $\mathrm{Re}(-1-\eta-\zeta)=\alpha+\beta-1<0$ in first and second terms of (46a), respectively. Substituting (45) and (46a) into (44), integrating over d^4k, and assuming $m_j^2 l^2 \ll 1$, we get the terms giving the contribution of weak interactions to the AMM of the leptons:

$$\tilde{\Gamma}_\mu^{(j)}(q)=\Gamma_\mu^{(j)}(p,q)|_{p^2=p'^2=m_j^2}=\frac{i}{2m_j}\sigma_{\mu\nu}q^\nu F(q^2)$$
$$a_j^{(w)}=F(0)=-\frac{G^2m_j^2}{(2\pi)^4 l^2}B \tag{46b}$$

Here the quantity B is given by

$$B=\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}\frac{1}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}\frac{\Gamma(2+\zeta)\Gamma(2+\eta)}{\Gamma(1-\zeta)\Gamma(1-\eta)}$$
$$\times\frac{v(-1-\eta-\zeta)}{\Gamma(4+\zeta+\eta)}\frac{\Gamma(-\eta-\zeta)}{\sin\pi(\eta+\zeta)}\left(\frac{1}{12}[(\eta+\zeta)(1+\eta+\zeta)+28]\right.$$
$$-\frac{2+\eta+\zeta}{60}\left\{8+(\eta+\zeta)\left[10+17(\eta+\zeta)+2(\eta+\zeta)^2\right]\right\}+\frac{2+\eta+\zeta}{1+\eta+\zeta}$$
$$\left.\times\left\{\frac{2+\eta+\zeta}{60(4+\eta+\zeta)}\left[-80+28(\eta+\zeta)+21(\eta+\zeta)^2+(\eta+\zeta)^3\right]\right\}\right) \tag{47}$$

An integral of the type (47) will be investigated in Sections 8.2 and 8.3 in detail (see also Appendix A). Shifting in turn the contour of integration to the right we can reduce this integral to the double series. The result of numerical calculations gives $B\approx 3$ for the form factor v_s.

Thus, we suppose, as before, that the obtained "weak" contribution (46b) is of an order not greater than the experimental error. This makes it possible to establish the following restrictions for the parameter l:

$$l\gtrsim 3\times 10^{-19}\ \text{cm} \qquad \text{for } a_e$$
$$l\gtrsim 1.1\times 10^{-17}\ \text{cm} \qquad \text{for } a_\mu$$

7.3.2. The Lamb Shift of Atomic Levels. If we restrict ourselves to an order of eG^2, as shown by Efimov et al. (1973), the dominant contribution to the Lamb shift due to weak interactions comes from the graphs (Figure

6b). The terms of the S matrix corresponding to these diagrams may be presented in the form

$$S^{(2,2)}(x,y) = -i : A_\mu(x)\Pi_{\mu\nu}(x-y)A_\nu(y):$$

where

$$\Pi_{\mu\nu}(x) = \frac{1}{(2\pi)^4}\int d^4q\, e^{iqx}\tilde{\Pi}_{\mu\nu}(q)$$

Calculating $\tilde{\Pi}_{\mu\nu}(q)$ we made use of expressions (43) and (46a) for $f^{(2,2)}_{\mu\nu\alpha\beta}(p,q,k)$ and $N_{\alpha\beta}(k)$, respectively. Thus, in the stochastic theory the quantity $\tilde{\Pi}_{\mu\nu}(q)$ has the form

$$\tilde{\Pi}_{\mu\nu}(q) = \frac{e^2G^2}{2(2\pi)^8}\iint d^4p\, d^4k\, \mathrm{Sp}\left\{ f^{(2,2)}_{\mu\nu\alpha\beta}(p,q,k)N_{\alpha\beta}(k)\right\}$$

$$= \frac{e^2G^2}{2(2\pi)^8}\iint d^4p\, d^4k\, \mathrm{Sp}$$

$$\times\Big\{\Big[\big(2S(\hat{p}+\hat{k})\big[d_\nu(-q)S^{-1}(\hat{p}+\hat{k}+\hat{q})\big]$$

$$\times S(\hat{p}+\hat{k}+\hat{q})\big[d_\mu(q)S^{-1}(\hat{p}+\hat{k})\big]S(\hat{p}+\hat{k})O_\alpha S(\hat{p})$$

$$-S(\hat{p}+\hat{k})\big\{d_\nu(-q)\big[d_\mu(q)S^{-1}(\hat{p}+\hat{k})\big]\big\}S(\hat{p}+\hat{k})O_\alpha S(\hat{p})$$

$$-S(\hat{p}+\hat{k})O_\alpha S(\hat{p})\big\{d_\nu(-q)\big[d_\mu(q)S^{-1}(\hat{p})\big]\big\}S(\hat{p})$$

$$+2S(\hat{p}+\hat{k})\big[d_\mu(q)S^{-1}(\hat{p}+\hat{k}-\hat{q})\big]S(\hat{p}+\hat{k}-\hat{q})O_\alpha S(\hat{p}-\hat{q})$$

$$\times\big[d_\nu(-q)S^{-1}(\hat{p})\big]S(\hat{p})+2S(\hat{p}+\hat{k})$$

$$\times O_\alpha S(\hat{p})\big[d_\nu(-q)S^{-1}(\hat{p}+\hat{q})\big]$$

$$\times S(\hat{p}+\hat{q})\big[d_\mu(q)S^{-1}(\hat{p})\big]S(\hat{p})\big)\Big]O_\beta N_{\alpha\beta}(k)\Big\}$$

Passing to the Euclidean metrics and using the generalization of Feynman's parametrization we get after some calculations

$$\tilde{\Pi}_{\mu\nu}(q) = \left(q_\nu q_\mu - q^2 g_{\mu\nu}\right)\Pi(q^2)$$

where

$$\Pi(q^2)=\frac{\delta}{\pi^2 i}\Gamma(-1-\eta-\rho-\zeta-\lambda)\int d^4p(p+q)^2[q^2+2(p\cdot q)]^{-1}$$

$$\times\left\{\left[\frac{m^2y}{1-x}+m^2(1-y)-(p+q)^2\right]^{1+\eta+\zeta+\rho+\lambda}\right.$$

$$\left.-\left[\frac{m^2y}{1-x}+m^2(1-y)-p^2\right]^{1+\eta+\zeta+\rho+\lambda}\right\} \tag{48}$$

Here

$$\delta=\frac{\alpha G^2}{8\pi^5}\frac{1}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}l^{2\zeta}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}l^{2\eta}$$

$$\times\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty}d\rho\frac{v(\rho)}{\sin\pi\rho}l^{2\rho}\frac{1}{2i}\int_{-\kappa+i\infty}^{-\kappa-i\infty}d\lambda\frac{v(\lambda)}{\sin\pi\lambda}l^{2\lambda}$$

$$\times\frac{\Gamma(2+\zeta)\Gamma(2+\eta)}{\Gamma(4+\eta+\zeta)\Gamma(1-\zeta)\Gamma(1-\eta)\Gamma(1-\rho)}\frac{1}{\Gamma(1-\lambda)\Gamma(-2-\eta-\zeta-\rho)}$$

$$\times\iint_0^1 dx\,dy(1-y)^{-\lambda}y^{-3-\eta-\zeta-\rho}\{[3\Gamma(-1-\eta-\zeta)-2\Gamma(-2-\eta-\zeta)$$

$$+x\Gamma(-2-\eta-\zeta)]\Gamma(-2-\eta-\zeta-\rho)$$

$$+x\Gamma(-2-\eta-\zeta)\Gamma(-1-\eta-\zeta-\rho)\}x^{2+\eta+\zeta}(1-x)^{\rho}$$

Using the identity (28), we obtain

$$\Pi(q^2)=\frac{1}{\pi^2 i}\delta\cdot\Gamma(-\eta-\zeta-\lambda-\rho)\int_0^1 dz\int d^4p(p+q)^2$$

$$\times\left[\frac{m^2y}{1-x}+m^2(1-y)-(p+qz)^2-q^2z(1-z)\right]^{\eta+\zeta+\lambda+\rho}$$

$$=\delta\int_0^1 dz\left[\frac{m^2y}{1-x}+m^2(1-y)-q^2z(1-z)\right]^{2+\eta+\zeta+\rho+\lambda}$$

$$\times\left\{-2\Gamma(-3-\eta-\zeta-\rho-\lambda)\left[\frac{m^2y}{1-x}+m^2(1-y)-q^2z(1-z)\right]\right.$$

$$\left.+q^2(1-z)^2\Gamma(-2-\eta-\zeta-\rho-\lambda)\right\}$$

The corrections to atomic levels produced by the nuclear field vacuum polarization are given by the following formula:

$$\Delta E = -\frac{8Z^4\alpha^2 m_e^2}{n_0^3}\mathrm{Ry}\frac{d\Pi(q^2)}{dq^2}\bigg|_{q^2=0} \tag{49}$$

where

$$\mathrm{Ry} = \tfrac{1}{2}\alpha^2 m_e$$

The calculation of $d\Pi(q^2)/dq^2|_{q^2=0}$ is performed easily. Assuming $m_j^2 l^2 \ll 1$, we get

$$\frac{d\Pi(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{2}{5}\frac{\alpha G^2}{\pi^5 l^4}\left(\frac{1}{m_\mu^2}+\frac{1}{m_e^2}\right)d \tag{50}$$

where

$$d = \frac{5}{48}\prod_{i=1}^{3}\left[\frac{1}{2i}\int_{-\gamma_i+i\infty}^{-\gamma_i-i\infty} dx_i \frac{v(x_i)}{\sin\pi x_i}\frac{1}{\Gamma(1-x_i)}\right]\frac{v(-2-x_1-x_2-x_3)}{\sin\pi(x_1+x_2+x_3)}$$

$$\times\frac{\Gamma(2+x_1)\Gamma(2+x_2)\Gamma(1+x_3)}{(3+x_1+x_2)\Gamma(5+x_1+x_2+x_3)}\Gamma(-2-x_1-x_2)\Gamma(-2-x_1-x_2-x_3)$$

$$\times\{3[8+3(x_1+x_2)]+(1+x_1+x_2+x_3)[9+x_3+3(x_1+x_2)]\}$$

Substituting (50) into (49) and taking into account that $m_\mu^2 \gg m_e^2$, we obtain the contribution to the Lamb shift for $2S_{1/2}-2P_{1/2}$ in the form

$$\Delta E_w(2S_{1/2}-2P_{1/2}) = -\frac{2}{5}\frac{Z^4}{\pi^5 l^4}\alpha^3 G^2 \mathrm{Ry}\, d \tag{51}$$

where the constant $d \sim 1$ for the form factor v_s (9).

The experimental value of the Lamb shift

$$\Delta E_{\exp}(2S_{1/2}-2P_{1/2}) = (1057.912 \pm 0.011)\ \mathrm{MHz}$$

due to the data analyzed by Brodsky and Drell (1970), Scadron (1980), is explained by local quantum electrodynamics. Therefore we get as above

$$l \gtrsim 2\times 10^{-16}\ \mathrm{cm}$$

8. SOME CONSEQUENCES OF NEUTRINO OSCILLATIONS IN THE NONLOCAL THEORY

8.1. Introduction. Many papers have appeared recently in which the problem of neutrino oscillations was considered within different approaches (see, for example, Bilenki and Pontecorvo, 1980a,b; Bilenki et al., 1980; Vuilleumier, 1979; Barger et al., 1980; for earlier work see the reviews of Bilenki and Pontecorvo, 1978; Wachsmuth, 1979; Morrison, 1980; and for a more popular presentation see Thomsen, 1980; Sutton, 1980; De Rujula and Glashow, 1980). The possibility of neutrino oscillations was first considered by Pontecorvo (1968): he assumed that the oscillation may appear if besides the usual weak interaction there is another interaction which does not conserve the lepton number. Such a picture is similar to the oscillation in the system of neutral kaons. Pontecorvo showed that massive neutrinos might change their identities during time evolution. A particle which is born as an electron neutrino in a beta decay may periodically behave as if it were a muon neutrino or a tau neutrino.

Many recent papers (see, for example, Cheng and Ling-Fong Li, 1980; Kang et al., 1980; Yanagida and Yoshimura, 1980, and references therein) devoted to the problem of neutrino oscillations deal with the unified theory of weak and electromagnetic interactions. Thus, in addition to the standard hypothesis of lepton-quark analogy, in these works a new conjecture is proposed that leptons, as the quarks, are mixed. In such a theory the oscillations $\nu_e \rightleftarrows \nu_\mu \rightleftarrows \nu_\tau$ appear.

Possible indications for neutrino oscillations have been obtained in the beam-dump experiments at CERN (De Rujula et al., 1980) and in the experiments with reactor antineutrinos (Barger et al., 1980; Reines et al., 1980). A number of experiments searching for neutrino oscillations stimulated recently interest to the question as they seem to give some indications for nonzero neutrino masses. A direct experiment of Lubimov and coworkers (1980) on measuring the $\tilde{\nu}_e$ mass from $3H$ decay gives the mass of the electron neutrino as between 14 and 46 eV, and most probably to be 36 ± 10 eV. The results of Reines et al. (1980) gave no direct implication for the neutrino mass, although they imply that the difference in mass between the two basis states lies in the region of 1 eV. It is worth noting that the recent developments of grand unified models nicely accomodates finite neutrino masses (Barbieri et al., 1980a,b; Gell-Mann et al., 1979; Georgi and Nanopoulos, 1979; Witten, 1980).

Thus we believe that a neutrino oscillation mechanism does exist in Nature and we will expect precise experiments on the properties of neutrinos.

Our aim here is modest and consists only in considering this problem from the viewpoint of stochasticity of space. We believe that due to the neutrino oscillation mechanism there exist mixed states of neutrinos which give nonorthogonality between neutrinos, say, ν_e and ν_μ, $\langle \nu_e | \nu_\mu \rangle \neq 0$ (ν_τ oscillation are not considered). In our previous paper (Dineykhan and Namsrai, 1975) such a possibility was considered and it was postulated there that the difference in behaviour between ν_e and ν_μ is caused by internal properties of these particles and depends of the nonlocal effects of weak interactions. For example,

$$\nu_\mu = \nu_e + f\nu = \nu_e - (1-\phi)\nu \tag{52}$$

where f (or ϕ) is some parameter which may be connected to a mixing angle and a value of mass difference of neutrino states, and ν is a massive neutrino (basic state) which possesses properties of both the ν_e and ν_μ neutrinos. In the representation (52) the transition propagator between ν_e and ν_μ has the form

$$\langle 0 | T\left(\Psi_{\nu_e}(x) \overline{\Psi}_{\nu_\mu}(y) \right) | 0 \rangle = \phi \mathfrak{D}_{\nu_e \nu}(x-y) \tag{53}$$

where we put $\mathfrak{D}_{\nu_e \nu_e}(x) = \mathfrak{D}_{\nu_e \nu}(x)$ since the ν neutrino possesses the property of ν_e.

If the neutrino mixing is assumed, the exotic decays $\mu \to e\gamma$, $\mu \to 3e$, $K^{\pm} \to \mu e \pi^{\pm}$, $K_L^0 \to \mu e$, etc., are in principle, possible. Notice that these decays are forbidden by the usual theory. These processes appear in higher orders of the perturbation theory. The present section is devoted to the investigation of these decays within the framework of our approach. It is shown here that if the parameter l for the weak interactions is of an order of $l \sim 10^{-16}$ cm and if the neutrino mixing takes place, then the probability for these decays is close to the experimental upper bounds (see, for example, Bricman et al., 1980; Fiorini, 1979). In this review, for example, we will consider the decays $\mu \to 3e$ and $K_L^0 \to \mu e$ in detail.

8.2. The $\mu \to 3e$ Decay. Within the framework of our model of weak interactions the probability decay is determined by diagram shown in Figure 7a. Before we proceed to estimating corrections from these diagrams to the probability of decays $\mu \to 3e$ and $K_L^0 \to \mu e$, let us consider diagrams of the type of neutrino–neutrino, neutrino–lepton, and lepton–lepton loops (Figure 8a, b, c). Here we investigate one of these diagrams, say, the neutrino–lepton loop corresponding to the diagrams shown in Figure 8b. Its

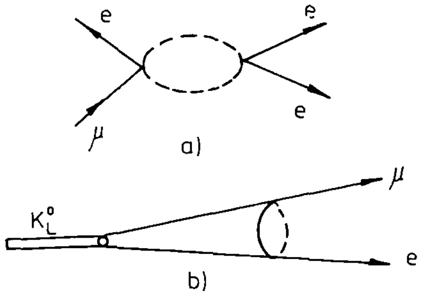

Fig. 7.

matrix element has the form

$$\Pi_{\alpha\beta}(p)=\frac{1}{(2\pi)^4 i}\int d^4k \mathrm{Sp}\{\hat{k}O_\alpha(m+\hat{k}+\hat{p})O_\beta\}$$

$$\times\frac{V_0(-k^2l^2)}{-k^2-i\varepsilon}\frac{V_m(-(k+p)^2l^2)}{m^2-(k+p)^2-i\varepsilon}$$

After the standard calculations we have

$$\Pi_{\alpha\beta}(p)=\frac{1}{2\pi^2}\frac{1}{2i}\int_{\substack{-\rho+i\infty\\(0<\rho<1)}}^{-\rho-i\infty}d\zeta\frac{v(\zeta)(m^2l^2)^\zeta}{\sin\pi\zeta}\frac{1}{2i}$$

$$\times\int_{\substack{-\gamma+i\infty\\(0<\gamma<1)}}^{-\gamma-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}(m^2l^2)^\eta\frac{1}{\Gamma(1-\eta)\Gamma(1-\zeta)}$$

$$\times\int_0^1 dx\left(\frac{1-x}{x}\right)^\zeta\left(1-\frac{p^2}{m^2}x\right)^{\eta+\zeta}$$

$$\times\left[\left(-2p_\alpha p_\beta+g_{\alpha\beta}p^2\right)x(1-x)\Gamma(-\eta-\zeta)\right.$$

$$\left.+g_{\alpha\beta}\Gamma(-1-\eta-\zeta)(m^2-p^2x)(1-x)\right] \qquad (54)$$

We notice that the neutrino–neutrino loop (46a) in Section 7 is obtained by substituting $m=0$ in expression (54).

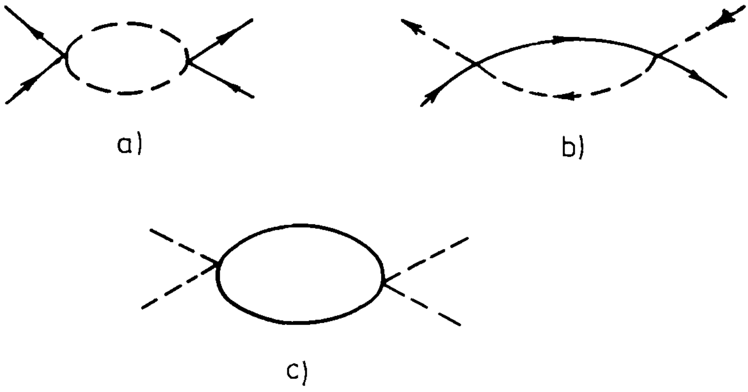

Fig. 8.

So, by definition (53) the matrix element corresponding the $\mu \to 3e$ decay has the form

$$M(\mu \to 3e) = \phi\left(\frac{G}{\sqrt{2}}\right)^2 (\bar{e}O_\alpha \mu) N_{\alpha\beta}(q)(\bar{e}O_\beta e)$$

where $N_{\alpha\beta}(q)$ is the neutrino–neutrino loop determined by (46a). Then the square of the matrix element equals

$$\sum_{\text{spin}} |M|^2 = \frac{G^4\phi^2c^2}{(\pi l)^4} 4[(k_1k_2)(k_0k_3) + (k_0k_2)(k_1k_3)] \tag{55a}$$

where

$$k_0 = k_1 + k_2 + k_3$$

$$c = -\frac{\pi}{2i}\int_{\substack{-\beta+i\infty \\ (0<\beta<1)}}^{-\beta-i\infty} d\zeta \frac{v(\zeta)v(-1-\zeta)}{\sin^2\pi\zeta} \tag{55b}$$

In the expression (55a) for the square of the matrix element we take into account main terms of the order of (G^2/l^2). After integration over phase space of the electrons and averaging over the muon spin we have

$$W(\mu \to 3e) = \frac{1}{192}\frac{G^4\phi^2c^2}{4l^4\pi^7} m_\mu^5$$

The probability branching ratio of $\mu \to 3e$ to $\mu \to e\nu\bar{\nu}$ constitutes

$$B(\mu \to 3e) = \frac{W(\mu \to 3e)}{W(\mu \to e\nu\bar{\nu})} = \frac{G^2\phi^2c^2}{4(\pi l)^4} \tag{56}$$

where

$$c = \begin{cases} 2,3 & \text{for} \quad v = v_s \\ 1,4 & \text{for} \quad v = v_b \\ 0,4 & \text{for} \quad v = v_1 \end{cases}$$

Here v_s, v_b, and v_1 are determined by formulas (9), (27), and (31), respectively, and $|\phi| \lesssim 0.055$ (see Lee et al., 1977). Assuming $\phi \sim 10^{-2}$ and $l \sim 2 \times 10^{-16}$ cm we get

$$B(\mu \to 3e) \lesssim \begin{cases} 1.35 \times 10^{-8} & \text{for } v_s \\ 4.9 \times 10^{-9} & \text{for } v_b \\ 3.9 \times 10^{-10} & \text{for } v_1 \end{cases} \tag{57}$$

8.3. The $K_L^0 \to \mu e$ Decay. In the second order in G the decay $K_L^0 \to \mu e$ is described by the diagram in Figure 7b. The term of the S matrix corresponding to this diagram can be rewritten in the form

$$i\sqrt{2}\, f_{KN\Lambda} \left(\frac{G}{\sqrt{2}} \right)^2 \bar{\mu}(p_-)\Gamma(p_-, q)e(p_+)\varphi_K \cos\theta_c \sin\theta_c \tag{58}$$

where

$$\Gamma(p_-, q) = \frac{\phi}{(2\pi)^4 i} \int d^4k\, \Pi^{\nu}_{\alpha\beta}(k) O_\alpha$$
$$\times \frac{m_N + \hat{k} + \hat{p}_+}{m_N^2 - (k + p_+)^2 - i\varepsilon} \gamma_5 \frac{m_N + \hat{k} - \hat{p}_-}{m_N^2 - (k - p_-)^2 - i\varepsilon} O_\beta$$
$$\times V_m\left(-(k + p_+)^2 l^2\right) V_m\left(-(k - p_-)^2 l^2\right) \qquad (m_N \sim m_\Lambda)$$

Here $\Pi^{\nu}_{\alpha\beta}(k)$ corresponds to the neutrino–nucleon loop considered in

Section 8.2. The hadron current is chosen in the Cabibbo form. After tedious but elementary calculations we get

$$\Gamma(p_-,q) = -m_N \hat{q}(1-\gamma_5)\frac{1}{16\pi^4 l^2}A$$

where

$$A = \frac{\pi}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} dy \frac{v(y)}{\sin \pi y}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\zeta \frac{v(\zeta)}{\sin \pi\zeta}\frac{1}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty} d\eta \frac{v(\eta)}{\sin \pi\eta}$$

$$\times \frac{v(-1-\eta-\zeta-y)}{\sin \pi(y+\eta+\zeta)}\frac{\Gamma(2+\zeta)\Gamma(2+\eta)\Gamma(-1-\eta-\zeta)}{\Gamma(1-\eta)\Gamma(1-\zeta)\Gamma(3+\eta+\zeta)} \tag{59}$$

As above, after displacement of the contours of integration to the right we obtain (see Appendix A)

$$A = 2\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\zeta \frac{v(\zeta)}{\sin^2 \pi\zeta}\frac{1}{\zeta(1+\zeta)(2+\zeta)}$$

$$\times \sum_{n=0}^{\infty}[v(n)v(-1-n-\zeta) - v(n-\zeta)v(-1-n)] \tag{60}$$

Now the matrix elements (58) acquires the form

$$M = i\sqrt{2} f_{KN\Lambda}\left(\frac{G}{\sqrt{2}}\right)^2 \cos\theta_c \sin\theta_c \phi \frac{m_N}{16\pi^4 l^2} A\varphi_K[\bar{\mu}(p_-)\hat{q}e(p_+)]$$

and its square is

$$|M|^2 = 2m_K^2\left(1-\frac{m_\mu^2}{m_K^2}\right)A^2 g^2$$

where

$$g = f_{KN\Lambda}\frac{G^2}{2}\cos\theta_c \sin\theta_c \phi \frac{m_\mu m_N}{16\pi^4 l^2}$$

Integration of the decay probability over the phase space of two leptons gives

$$W(K_L^0 \to \mu e) = \frac{m_K}{8}\left(1-\frac{m_\mu^2}{m_K^2}\right)^2 A^2 \cos^2\theta_c \sin^2\theta_c \frac{\phi^2 G^4}{l^4}\frac{f_{KN\Lambda}^2}{4\pi}\frac{m_\mu^2 m_N^2}{128\pi^8} \tag{61}$$

where $\sin\theta_c = 0.23$, $f_{KN\Lambda}^2/4\pi = 10.0 \pm 2.8$ (see, for example, Ebel, 1970; Nagels et al., 1976), $\phi \sim 10^{-2}$, and $l \sim 2\times 10^{-16}$ cm;

$$A = \begin{cases} -1.8 & \text{for} \quad v = v_s \\ -2.7\times 10^{-1} & \text{for} \quad v = v_b \\ -2.10^{-2} & \text{for} \quad v = v_1 \end{cases} \tag{62}$$

Thus, the branching ratio of this exotic decay in our model is

$$B = \frac{W(K_L^0 \to \mu e)}{W(K_L^0 \to \text{all})} \lesssim \begin{cases} 1.2\times 10^{-8} & \text{for} \quad v = v_s \\ 1.4\times 10^{-10} & \text{for} \quad v = v_b \\ 7.4\times 10^{-13} & \text{for} \quad v = v_1 \end{cases}$$

We see that the exotic decays $\mu \to 3e$ and $K_L^0 \to \mu e$ depend strongly from the form factors of the nonlocal theory. Our results together with the calculations by Cheng and Ling-Fong Li (1977) and the experimental upper bounds [Bricman et al., 1980 (Particle Data Group)] are summarized in Table I. These numerical calculations are presented for a purely illustrative purpose. These are important in a sense that they permit one to estimate a parameter of mixing and the value of elementary length l. The introduced parameter ϕ is connected to the mixing angle and the mass difference of neutrinos ν_1 and ν_2 (or heavy leptons N_1 and N_2) of other models of weak interactions by the formula

$$\phi \sim \sin\varphi \cos\varphi \Delta m_i \qquad (i = \nu_i, N_i)$$

TABLE I

Process	Our calculations		Results of Cheng and Li	Experimental upper limits
$\mu \to 3e$	1.3×10^{-8}	$v = v_s$	10^{-12}	1.9×10^{-9}
	4.9×10^{-9}	$v = v_b$		
	3.9×10^{-10}	$v = v_1$		
$K_L^0 \to \mu e$	1.2×10^{-8}	$v = v_s$	10^{-10}	2×10^{-9}
	1.4×10^{-10}	$v = v_b$		
	7.4×10^{-13}	$v = v_1$		
$K_L^0 \to \mu^+\mu^-$	1.2×10^{-6}	$v = v_b$	—	$(9.1\pm 1.9)\times 10^{-9}$
	6.7×10^{-9}	$v = v_1$		
$\Delta m(K_L^0 - K_S^0)$	$5.10^{11}\hbar\ \text{sec}^{-1}$	$v = v_b$	—	$0.5\times 10^{10}\hbar\ \text{sec}^{-1}$
	$1.10^{11}\hbar\ \text{sec}^{-1}$	$v = v_1$		

9. NEUTRINO ELECTROMAGNETIC PROPERTIES IN NONLOCAL (STOCHASTIC) THEORY OF WEAK INTERACTIONS

In the last years great attention has been paid to the physical properties of neutrinos. This is connected with the problem of neutrino oscillations and its mass, and astrophysical consequences (Faver et al., 1978; Schramm, 1979; Schramm and Steigman, 1980; this problem has been discussed by many authors at the Neutrino 1979 Conference), and also with the rapid development of neutrino experiments carried out at CERN, Caltech-FNAL, Serpukhov, etc. (see, for example, Busser, 1980; Winter, 1979; Baltay, 1979; Arbuzov, 1975).

As usual, the neutrino is considered as a weak-interacting particle with zero mass and without an electric charge. Therefore among the electromagnetic characteristics of neutrinos the only nonvanishing quantity is its charged radius r_ν. Possible experiments on measuring the charge radius of the neutrinos have been pointed out by Andryushin et al., 1971.

However, it seems that the neutrino mass is not zero (see Section 8). Then the neutrino may possess the magnetic moment a_ν. Recently, magnetic moment of a massive neutrino has been discussed by Fujikawa and Shrock (1980).

Starting, with the analysis of experimental data on inclusive reactions $\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + \text{anything}$ and $\nu_\mu - e$ elastic scattering, Bardin and Mogilevski (1974), Kim et al. (1974), and Arbuzov (1974) have investigated the electromagnetic properties of neutrinos and calculated the correction to the cross sections of these reactions due to Feynman diagrams involved in one-photon exchange, and obtained restriction on the charged radius and magnetic moment of the neutrino.

The contribution due to the one-photon exchange calculated by these authors is called electromagnetic, although, as is known, the charged radius and magnetic moment of the neutrino must appear due to effects of the weak interactions. The calculation of these quantities in the usual theory of weak interactions meets difficulties because of divergences in the S-matrix elements.

The present section is devoted to the calculation of contributions to r_ν and a_ν within the framework of our approach formulated in this paper. In Feynman diagrams of the order eG^2 giving the corrections to r_ν and a_ν there are closed loops constructed by propagators of the charged leptons and neutrinos for calculations of which it is necessary to apply the method of the stochastic theory.

In the nonlocal (stochastic) theory of weak interactions the electromagnetic interaction of the neutrino, say, the muon one, in the lowest order of G is given by the following Feynman diagrams (Figure 9):

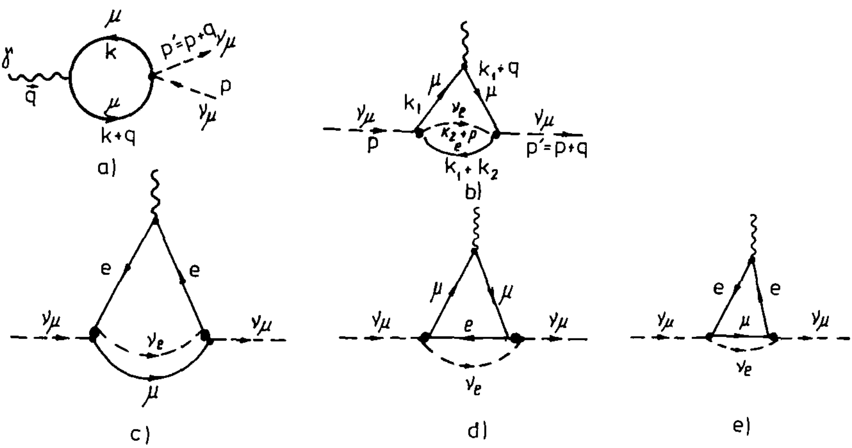

Fig. 9.

The matrix elements of the vertex functions corresponding to the diagrams shown in Figure 9 between two-neutrino states which are used for the calculations of the electromagnetic form factors of neutrino have the following general structure:

$$M = ie\bar{u}_\nu(p')\{\gamma_\mu F_1(q^2) + i\sigma_{\mu\rho}q^\rho F_2(q^2)\}u_\nu(p) \tag{63}$$

where

$$F_1(q^2) = \frac{1}{6}r_\nu^2 q^2, \qquad F_2(0) = \frac{1}{2m_e}a_\nu, \qquad q = p' - p$$

Here r_ν^2 is the mean-square charged radius of the neutrino and a_ν is its magnetic moment in the units of electron Bohr magnetons.

In order to calculate the contributions from the diagrams shown in Figure 9 to r_ν and a_ν we shall investigate them separately. First, let us consider the lepton loop (Figure 9a). The term for the S-matrix corresponding to this diagram has the form

$$M_1 = ie\frac{G}{\sqrt{2}}\bar{u}_\nu(p')O_\alpha u_\nu(p)K_{\alpha\beta}(q)A^\beta(q) \tag{64}$$

where the $K_{\alpha\beta}(q)$ in the stochastic theory is given by

$$K_{\alpha\beta}(q) = \int \frac{d^4k}{(2\pi)^4 i}\mathrm{Sp}\{O_\alpha S(\hat{k}+\hat{q})U_\beta(q,k)S(\hat{k})\}$$

Here

$$U_\beta(k,q) = -d_\beta(q)S^{-1}(\hat{k})$$

On the other hand, taking into account the identity (17) for the d operation we have

$$K_{\alpha\beta}(q) = \int \frac{d^4k}{(2\pi)^4 i} \mathrm{Sp}\{O_\alpha d_\beta(q) S(\hat{k})\}$$

By definition

$$d_\beta(q)S(\hat{k}) = \frac{1}{m-\hat{k}-\hat{q}}\gamma_\beta \frac{V_m(-k^2l^2)}{m-\hat{k}} + \frac{1}{m-\hat{k}-\hat{q}} \times \left[V(-(k+q)^2l^2) - V(-k^2l^2)\right]\frac{\hat{q}\gamma_\beta}{q^2}$$

After some elementary calculations we obtain the gauge-invariant expression for $K_{\alpha\beta}(q)$:

$$K_{\alpha\beta}(q) = \frac{1}{2\pi^2}\left(q_\alpha q_\beta - q^2 g_{\alpha\beta}\right)\tilde{K}(q^2)$$

where

$$\tilde{K}(q^2) = -\frac{1}{2i}\int_{\substack{-\alpha+i\infty \\ (0<\alpha<1)}}^{-\alpha-i\infty} d\zeta \frac{v(\zeta)}{\sin\pi\zeta}(m^2l^2)^\zeta \frac{\Gamma(-\zeta)}{\Gamma(1-\zeta)} \times \int_0^1 dx (1-x)^{1-\zeta} x \left[1 - \frac{q^2}{m^2}x(1-x)\right]^\zeta$$

Making use of the identity

$$\Gamma(-\zeta) = -\pi/\sin\pi\zeta\,\Gamma(1+\zeta)$$

we have

$$\tilde{K}(q^2) = \frac{\pi}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty} d\zeta \frac{v(\zeta)}{[\sin\pi\zeta]^2} \frac{1}{\Gamma(1-\zeta)\Gamma(1+\zeta)} \times \int_0^1 dx(1-x)^{1-\zeta} x\left(1 - \frac{q^2}{m^2}x(1-x)\right)^\zeta$$

$$= \int_0^1 dx(1-x)x\left[v'(0) + \log m^2l^2 + \log\frac{m^2 - q^2x(1-x)}{m^2(1-x)}\right]$$

Here

$$\int_0^1 dx\, x(1-x)\log(1-x)=\frac{\Gamma(2)\Gamma(2)}{\Gamma(4)}[\psi(2)-\psi(4)]=-\frac{5}{36}$$

The first diagram considered gives a contribution to the mean-square radius of the neutrino only:

$$r_{1\nu}^2=6\frac{\partial F_1^{(1)}(q^2)}{\partial q^2}\bigg|_{q^2=0}=\frac{G}{\sqrt{2}}\frac{1}{2\pi^2}\left[\frac{5}{6}-v'(0)-\log m^2 l^2\right] \qquad (65)$$

Assuming, as above, $l \sim 2\times 10^{-16}$ cm we get

$$r_{1\nu}^2=\begin{cases} 1\times 10^{-33}\ \text{cm}^2 & \text{for} \quad v=v_s \\ 0.9\times 10^{-33}\ \text{cm}^2 & \text{for} \quad v=v_b \\ 1.1\times 10^{-33}\ \text{cm}^2 & \text{for} \quad v=v_1 \end{cases}$$

We see that the contribution from the diagram (Figure 9a) to the mean-square radius of the neutrino is of the order of 10^{-33} cm^2.

Now we turn to discussion of the vertex diagrams. There are only eight diagrams for each neutrino ν_i $(i=e,\mu)$. Among them there exist diagrams of different structure, for example, diagrams, shown in Figure 8b–e with the calculating of which we shall start. The terms for the S matrix corresponding to these diagrams are

$$\begin{aligned}
\Gamma_{1\rho}(p,q)&=N\cdot O_\alpha\left[d_\rho(-q)S^{(\mu)}(\hat{k}_1+\hat{p})\right] \\
&\quad\times O_\beta \mathrm{Sp}\left\{O_\alpha S^{(e)}(\hat{k}_1+\hat{k}_2)O_\beta S^{(\nu)}(\hat{k}_2)\right\} \\
\Gamma_{2\rho}(p,q)&=N\cdot O_\alpha S^{(\mu)}(\hat{k}_1+\hat{p}) \\
&\quad\times O_\beta \mathrm{Sp}\left\{O_\alpha\left[d_\rho(-q)S^{(e)}(\hat{k}_1+\hat{k}_2)\right]O_\beta S^{(\nu)}(\hat{k}_2-\hat{q})\right\} \\
\Gamma_{3\rho}(p,q)&=N\cdot O_\alpha S^{(\nu)}(\hat{k}_2-\hat{q}) \\
&\quad\times O_\beta \mathrm{Sp}\left\{O_\alpha\left[d_\rho(-q)S^{(e)}(\hat{k}_1+\hat{k}_2)\right]O_\beta S^{(\mu)}(\hat{k}_1+\hat{p})\right\} \\
\Gamma_{4\rho}(p,q)&=N\cdot O_\alpha S^{(\nu)}(\hat{k}_2)O_\beta \mathrm{Sp}\left\{O_\alpha S^{(e)}(\hat{k}_1+\hat{k}_2)O_\beta\left[d_\rho(-q)S^{(\mu)}(\hat{k}_1+\hat{p})\right]\right\} \\
N&=\int\frac{d^4k_1}{(2\pi)^4 i}\int\frac{d^4k_2}{(2\pi)^4 i}
\end{aligned} \qquad (66)$$

respectively.

Let us consider the first expression of (66). We are interested in those terms only which give contributions to r_ν and a_ν in the limit $q^2 \to 0$. The structure of the type (63) is obtained by the usual way. After tedious calculations we have

$$\Gamma_{1\rho}(p,q) = \frac{1}{16\pi^4 l^2}\frac{1}{3}\left[\gamma_\rho q^2 F_1^{(1)} + i\sigma_{\rho\alpha}q^\alpha m_\nu F_2^{(1)}\right] \tag{67}$$

where

$$F_1^{(1)} = \rho(\zeta,\eta)\{2(3+2\eta+2\zeta)\Gamma(-1-\eta-\zeta)+\Gamma(-\eta-\zeta)[\tfrac{9}{2}-(1+\eta+\zeta)]\}$$

$$F_2^{(1)} = \rho(\zeta,\eta)\Gamma(-\eta-\zeta)\left[-\frac{1+\eta+\zeta}{2}(1+2\eta+2\zeta)-13-6\eta-6\zeta\right]$$

Here

$$\rho(\zeta,\eta) = \frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\zeta\frac{v(\zeta)}{\sin\pi\zeta}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty} d\eta\frac{v(\eta)}{\sin\pi\eta}\frac{v(-1-\eta-\zeta)}{\sin\pi(\eta+\zeta)}$$

$$\times\frac{\Gamma(2+\eta)\Gamma(2+\zeta)}{\Gamma(1-\eta)\Gamma(1-\zeta)}\frac{1}{\Gamma(4+\eta+\zeta)}$$

Similar calculations give the following structures for $\Gamma_{4\rho}(p,q)$ and $\Gamma_{2\rho}(p,q)$:

$$\Gamma_{4\rho}(p,q) = \frac{1}{16\pi^4 l^2}\frac{1}{6}\left[\gamma_\rho q^2 F_1^{(4)} + i\sigma_{\rho\alpha}q^\alpha m_\nu F_2^{(4)}\right] \tag{68}$$

where

$$F_1^{(4)} = \rho(\zeta,\eta)\{[-2+2(1+\eta+\zeta)]\Gamma(-\eta-\zeta)-10\Gamma(-1-\eta-\zeta)\}$$

$$F_2^{(4)} = \rho(\zeta,\eta)\left[-10-17(1+\eta+\zeta)+4(1+\eta+\zeta)^2\right]\Gamma(-\eta-\zeta)$$

and

$$\Gamma_{2\rho}(p,q) = \frac{1}{16\pi^4 l^2}\left[\gamma_\rho q^2 F_1^{(2)} + i\sigma_{\rho\alpha}q^\alpha m_\nu F_2^{(2)}\right] \tag{69}$$

Here

$$F_1^{(2)} = \rho(\zeta,\eta)\Gamma(1-\eta-\zeta)$$

$$\times\Bigg\{\frac{3+x}{(1-\eta)(1+\eta)}\left[\frac{x-1}{6}-\frac{2+x}{24}(3+x)-\frac{5}{2}\frac{2+x}{\eta}\right]$$

$$+\frac{1}{2-\eta}\left[\frac{2+\zeta}{1+\eta}\left(-\frac{7}{6}-\frac{5}{6}x+\frac{5}{2}\frac{3+x}{\eta}\right)\right.$$

$$\left.-\frac{(1-x)(4+x)}{12}+\frac{1}{3}\frac{3+x}{1+\eta}(1-4x)\right]+\frac{1-x}{3(3-\eta)}\left(1-\frac{2+\zeta}{1+\eta}\right)$$

$$+\frac{1}{(1-\eta)(2-\eta)}\left[\frac{1-x}{3}-\frac{5}{2}\frac{2+x}{1+\eta}(3+x)\right]+\frac{1}{(2-\eta)(3-\eta)}$$

$$\times\left[\frac{3+x}{1+\eta}\left(-\frac{41}{6}-\frac{25}{6}x\right)+\frac{2+\zeta}{1+\eta}\left(-1-\frac{3}{2}x+\frac{5}{2}\frac{3+x}{\eta}\right)\right.$$

$$\left.+(2+x)\left(\frac{11}{6}+\frac{5}{3}x\right)+\frac{(2-x)(1-x)}{12}\right]+\frac{\eta}{(1-\eta)(2-\eta)}$$

$$\times\left[-\frac{1}{2}\frac{3+x}{1+\eta}(1-x)+(2+x)\left(\frac{17}{24}+\frac{7}{24}x\right)+\frac{1}{2}\frac{2+\zeta}{1+\eta}\right]$$

$$+\frac{1}{(1-\eta)(2-\eta)(3-\eta)}\frac{2}{3}\frac{2+\zeta}{1+\eta}(2+x)$$

$$+\frac{2+x}{3}\frac{3+x}{1+\eta}+\frac{\eta}{(2-\eta)(3-\eta)}$$

$$\times\left[(2+x)\left(\frac{1}{6}-\frac{x}{4}+\frac{5}{6}\frac{2+\zeta}{1+\eta}\right)+\frac{3+x}{1+\eta}\left(-\frac{1}{2}+x\right)\right]\Bigg\}$$

$$F_2^{(2)} = \frac{\rho(\zeta,\eta)\Gamma(1-\eta-\zeta)}{6x(1-\eta^2)(2-\eta)}\left[20x(5x+6+x^2)+9\eta(1-\eta)+2\eta x^2(1-x)\right.$$

$$+18x\eta(1-\eta)$$

$$+\eta x^2(\zeta-\eta)-8x\eta^2(1+x)-4\eta^2x^2(1+x)$$

$$\left.-7x\eta^2(x+\eta)-\eta^2x^2(3+2\zeta)\right]$$

$$(x=\eta+\zeta)$$

It is seen easily that by substitution of the variables of integration the third term can be transformed to the second term of the expression (66). Thus, in our case of $q^2 \to 0$ we get

$$\Gamma_{3\rho}(p,q)=\Gamma_{2\rho}(p,q)$$

Now we carry out the numerical calculations for the concrete form of the function $v(\zeta)$, say, for v_s, which is determined by (9). The results of a displacement of the contours of integration in (67)–(69) give

$$\begin{aligned} F_1^{(1)} &= -0.18, & F_2^{(1)} &= 2.56 \\ F_1^{(4)} &= -0.28, & F_2^{(4)} &= 6.7 \\ F_1^{(2)} &= -0.05, & F_2^{(2)} &= 1.07 \end{aligned} \tag{70}$$

Finally, we shall consider the expressions (67), (68), and (69) together with (70). Then

$$M_\rho = \delta(\Gamma_{1\rho}+2\Gamma_{2\rho}+\Gamma_{4\rho}) = \delta\left[\gamma_\rho F_1(q^2)+i\sigma_{\rho\alpha}q^\alpha m_\nu F_2(q^2)\right] \tag{71}$$

where

$$F_2(0)=\frac{1}{16\pi^4 l^2}(f_1+2f_2+f_4), \qquad \delta = ie\frac{G^2}{2}$$

$$F_1(q^2)=\frac{q^2}{16\pi^4 l^2}(R_1+2R_2+R_4)$$

Here

$$f_1=\tfrac{1}{3}F_2^{(1)}(0)=2.56/3, \qquad f_2=F_2^{(2)}(0)=1.07$$

$$f_4=\tfrac{1}{6}F_2^{(4)}(0)=1.1$$

$$R_1=\tfrac{1}{3}(-0.18), \qquad R_4=\tfrac{1}{6}(-0.28), \qquad R_2=-0.05$$

Therefore

$$r_{2\nu}^2=\langle r_{2\nu}^2\rangle=\frac{6}{q^2}F_1(q^2)G^2$$

$$a_{\nu_j}=4m_{\nu_j}m_e\frac{G^2}{2}F_2(0), \qquad \nu_j=\nu_e,\nu_\mu$$

in the units of electron Bohr magneton.

The recent experiments and the analysis of the data [see, for example, Bardin and Mogilevski, 1974; Kim et al., 1974; Arbuzov, 1974; Daum et al., 1978; Bricman et al., 1980 (Particle Group Data)] establish the following restrictions for a_ν, $\langle r_\nu^2 \rangle$, m_{ν_μ}, and m_{ν_e}

$$|a_\nu| \lesssim 10^{-7}, \qquad m_{\nu_\mu} \lesssim 0.57 \text{ MeV}$$

$$\langle r_\nu^2 \rangle \lesssim 10^{-31} \text{ cm}^2, \qquad m_{\nu_e} \lesssim 6 \times 10^{-5} \text{ MeV}$$

Then, our result gives the following restrictions:

$$|a_{\nu_\mu}| \lesssim 2 \times 10^{-15}$$

$$|a_{\nu_e}| \lesssim 1.75 \times 10^{-19}$$

and

$$\langle r_\nu^2 \rangle \lesssim 1 \times 10^{-33} \text{ cm}^2$$

in the assumption $l \sim 2 \times 10^{-16}$ cm. If $(14 \leqslant m_{\nu_e} \leqslant 46)$ eV due to Lubimov et al. (1980), then

$$4 \times 10^{-20} \lesssim |a_{\nu_e}| \leqslant 1.34 \times 10^{-19}$$

We notice that in our model the vertex diagrams, shown in Figure 9b–e, give small contributions to $\langle r_\nu^2 \rangle$ of an order $(10^{-37}\text{–}10^{-38})$ cm^2 with respect to the diagram 9a.

10. STUDIES OF THE DECAY $K_L^0 \to \mu^+\mu^-$ AND K_L^0- AND K_S^0-MESON MASS DIFFERENCE WITHIN THE NONLOCAL (STOCHASTIC) THEORY OF WEAK INTERACTIONS

10.1. Introduction. Some time ago the rare decay $K_L^0 \to \mu^+\mu^-$ has been observed [see Bricman et al. (Particle Data Group), 1980], branching ratio of which coincides, in order of magnitude, with the unitary limit (Sehgal, 1969; Quigg and Jackson, 1968)

$$B_u(K_L^0 \to \mu^+\mu^-) = \frac{W(K_L^0 \to \mu^+\mu^-)}{W(K_L^0 \to \text{all})} \gtrsim 6 \times 10^{-9}$$

This process occurs essentially because of electromagnetic interactions (through two-photon exchange). It is interesting to notice that the well-known so-called GIM mechanism (the hypothesis of existence of the charm quark) started from the investigation of this process within the standard model of electroweak interactions (Glashow et al., 1970).

The calculation of "weak" corrections to the $B(K_L^0 \to \mu^+\mu^-)$ and the mass difference of K_L^0 and K_S^0 mesons within the usual nonrenormalizable theory (the four-fermion theory and the theory with intermediate bosons) of weak interactions gives a very small value of the cutoff momentum Λ of an order of few GeV. This contradicts the value of the natural cutoff $\Lambda \sim 10^2 - 10^3$ GeV for the growth of weak interactions (see, for example, Ioffe, 1973).

In this section we shall show the problem of suppression of the order $O(G^2)$ in decay $K_L^0 \to \mu^+\mu^-$ and mass difference $\Delta m(K_L^0 - K_S^0)$ may be solved within the framework of the stochastic (nonlocal) theory of weak interactions without introducing the fourth quark.

In our model contributions of nonlocal interactions to the $K_L^0 \to \mu^+\mu^-$ and the $\Delta m(K_L^0 - K_S^0)$ arise from the diagrams, shown in Figure 10.

10.2. The $K_L^0 \to \mu^+\mu^-$ Decay. $K_L^0 \to \mu^+\mu^-$ decay in the second order in G is described by the diagram shown in Figure 10a. The corresponding term has the form

$$M(K_L^0 \to \mu^+\mu^-) = i\sqrt{2}\, f_{KN\Lambda}\left(\frac{G}{\sqrt{2}}\right)^2 \bar{\mu}(p_-)\Gamma(p_-, q)\mu(p_+)\varphi_K \cos\theta_c \sin\theta_c$$

where $\Gamma(p_-, q)$ was calculated in section 8.3. Thus the branching ratio of

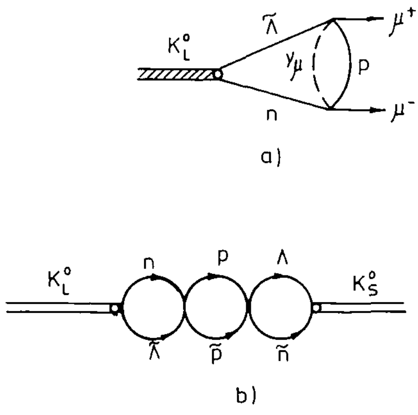

Fig. 10.

this decay is

$$B(K_L^0 \to \mu^+\mu^-) = \frac{W(K_L^0 \to \mu^+\mu^-)}{W(K_L^0 \to \text{all})} = 1.7A^2 \times 10^{-5} \tag{72}$$

where

$$W(K_L^0 \to \mu^+\mu^-) = \frac{m_K}{8}\left(1 - \frac{2m_\mu^2}{m_K^2}\right)^2 A^2 \cos^2\theta_c \sin^2\theta_c \frac{G^4}{l^4}\frac{m_N^2 m_\mu^2}{128\pi^8}\frac{f_{KN\Lambda}^2}{4\pi}$$

and A is given by (59) and (60). Substitution of numerical value (62) for A into (72) leads to the contribution

$$B(K_L^0 \to \mu^+\mu^-) = \begin{cases} 1.25 \times 10^{-6} & \text{for} \quad v = v_h \\ 6.7 \times 10^{-9} & \text{for} \quad v = v_l \end{cases} \tag{73}$$

10.3. The Mass Difference of K_L^0 and K_S^0 Mesons. Let us find now the energy operator for the transition K_L^0 into K_S^0. A typical diagram of the order G^2 giving the contribution to the $\Delta m(K_L^0 - K_S^0)$ is shown in Figure 10b. The expression corresponding to this diagram is

$$\tilde{\Sigma}(p) = G^2 \sin^2\theta_c \cos^2\theta_c \Pi_\alpha(p)\Pi_{\alpha\beta}(p)\Pi_\beta(p) \tag{74}$$

where

$$\Pi_\alpha(p) = f_{KN\Lambda}\int \frac{d^4k}{(2\pi)^4 i}\mathrm{Sp}\{S(\hat{k})O_\alpha S(\hat{k}+\hat{p})\gamma_5\}, \qquad S \equiv S_R$$

and

$$\Pi_{\alpha\beta}(p) = \int \frac{d^4k}{(2\pi)^4 i}\mathrm{Sp}\{S(\hat{k})O_\alpha S(\hat{k}+\hat{p})O_\beta\}$$

which is determined by the expression of type (54). In the case $m_K^2 l^2 \ll 1$ the expression obtained for $\Pi_{\alpha\beta}(p)$ on the mass shell of the K-meson acquires the form

$$\Pi_{\alpha\beta}(p) = g_{\alpha\beta}\frac{m_K^2}{m_K^2 l^2}\frac{1}{4\pi^2}c \tag{75}$$

where c is given by (55b).

The calculation of functions $\Pi_j(p)$ $(j=\alpha,\beta)$ is similar. As a result

$$\Pi_j(p)=\frac{f_{KN\Lambda}m_N p_j}{4\pi^2}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}l^{2\zeta}\frac{1}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}l^{2\eta}$$
$$\times\frac{\Gamma(-\eta-\zeta)}{\Gamma(1-\eta)\Gamma(1-\zeta)}\int_0^1 dx\, x^{-\eta}(1-x)^{-\zeta}\left[m_N^2-p^2x(1-x)\right]^{\eta+\zeta}$$

The contour integration gives

$$\Pi_j(p)=-f_{KN\Lambda}\frac{m_N p_j}{4\pi^2}\left\{v'(0)+1+\log m_N^2 l^2+\int_0^1 dx\log\left[1-\frac{p^2}{m_N^2}x(1-x)\right]\right.$$
$$\left.-\frac{\pi}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)v(-\zeta)}{\sin^2\pi\zeta}\right\} \tag{76}$$

Here it is necessary to carry out renormalization in the strong coupling constant $f_{KN\Lambda}$. After such renormalization the expression (76) acquires the form

$$\Pi_j(p)=-\frac{m_N p_j}{4\pi^2}f_{KN\Lambda}\rho$$
$$\rho=\int_0^1 dx\log\left[1-\frac{p^2}{m_N^2}x(1-x)\right] \tag{77}$$

Substituting (77) and (75) into (74) we obtain the following expression for $\Delta m(K_L^0-K_s^0)$:

$$\Delta m\left(K_L^0-K_S^0\right)=\frac{f_{KN\Lambda}^2}{(2\pi)^6}G^2\sin^2\theta_c\cos^2\theta_c m_N^2\frac{c\rho^2}{l^2}m_K \tag{78}$$

or

$$\Delta m\left(K_L^0-K_S^0\right)=\begin{cases}1\times10^{12}\hbar\ \sec^{-1} & \text{for}\quad v=v_s\\ 5\times10^{11}\hbar\ \sec^{-1} & \text{for}\quad v=v_b\\ 1\times10^{11}\hbar\ \sec^{-1} & \text{for}\quad v=v_1\end{cases} \tag{79}$$

We see that in the case of our choice of form factors v_s, v_b and v_1 the contributions (79) calculated are large with respect to the experimental value of $\Delta m_{\mathrm{exp}}=0.5\times 10^{10}\hbar$ sec^{-1}. However, by some other choice of form factors this inessential contradiction between the theoretical calculations and experimental data for $\Delta m(K_L^0-K_S^0)$ in our model may be easily eliminated. Table I presents the contributions calculated within the nonlocal theory and experimental data on $K_L^0\to\mu^+\mu^-$ and the mass difference $\Delta m(K_L^0-K_S^0)$ for different form factors of the theory. It should be noticed that in the real physical processes, apart from the value of elementary length, an important role may be played by the form factor of the theory.

Therefore, the problem of suppression of rare decays ($K_L^0\to\mu^+\mu^-$, $K^+\to\pi^+\nu\tilde{\nu}$, etc.) connected with the neutral currents $\Delta S=1$ and also calculations of the physical quantities of the order G^2 (for example, the mass difference of K_L^0 and K_S^0 mesons) may be solved within the framework of the nonlocal (stochastic) theory of quantized fields.

11. APPENDIX A. CALCULATION OF THE CONTOUR INTEGRAL

In this appendix we give the method of calculation of the contour integral, say, for example,

$$A=\frac{\pi}{2i}\int_{-\gamma+i\infty}^{-\gamma-i\infty}dy\frac{v(y)}{\sin\pi y}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\zeta\frac{v(\zeta)}{\sin\pi\zeta}\frac{1}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\eta\frac{v(\eta)}{\sin\pi\eta}$$
$$(0<\alpha,\beta,\gamma<1)$$
$$\times\frac{v(-1-\eta-y-\zeta)}{\sin\pi(y+\eta+\zeta)}\frac{\Gamma(2+\zeta)\Gamma(2+\eta)}{\Gamma(1-\eta)\Gamma(1-\zeta)}\frac{\Gamma(-1-\eta-\zeta)}{\Gamma(3+\eta+\zeta)}\tag{A1}$$

First we displace the contour γ to the right. Then the poles will appear at points $y=0,1,2,3,\ldots$ and $y=n-\eta-\zeta$ $(n=0,1,2,\ldots)$. In the first case $(y=0,1,2,\ldots)$ it is necessary to displace one of the other contours, say, β contour. The calculations of residues at points $\zeta=0,1,2,\ldots$ and $\zeta=N-\eta$ $(N=0,1,\ldots)$ give

$$A=2\frac{\pi}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\eta\frac{v(\eta)}{\sin^2\pi\eta}\frac{1}{\eta(1+\eta)(2+\eta)}\sum_{k=0}^{\infty}v(k)v(-1-k-\eta)\tag{A2}$$

After similar calculations of residues at points $\zeta=0,1,2,\ldots$ and $\zeta=N-\eta$ in the second case ($y=n-\eta-\zeta$) we get

$$A=-2\frac{\pi}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\eta\frac{v(\eta)}{\sin^2\pi\eta}\frac{1}{\eta(1+\eta)(2+\eta)}\sum_{k=0}^{\infty}v(k-\eta)v(-1-k) \tag{A3}$$

We unify now the expressions (A2) and (A3). Then

$$A=\frac{2\pi}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\eta\frac{v(\eta)}{\sin^2\pi\eta}\frac{1}{\eta(1+\eta)(2+\eta)}$$
$$\times\sum_{k=0}^{\infty}[v(k)v(-1-k-\eta)-v(k-\eta)v(-1-k)] \tag{A4}$$

The last integral (A4) is calculated easily for the concrete form of the form factor of the theory. For example, let

$$v=v_b=2^{1+2\zeta}/\Gamma(3+2\zeta)$$

Then

$$\sum_{k=0}^{\infty}[v(k)v(-1-k-\eta)-v(k-\eta)v(-1-k)]$$
$$=2^{-2\eta}\left[\frac{1}{\Gamma(3)\Gamma(1-2\eta)}+\frac{1}{\Gamma(5)\Gamma(-1-2\eta)}\right.$$
$$\left.+\frac{1}{\Gamma(7)\Gamma(-3-2\eta)}+\cdots-\frac{1}{\Gamma(3-2\eta)}\right]$$

and

$$A=4\frac{\pi}{2i}\int_{-\alpha+i\infty}^{-\alpha-i\infty}d\eta\frac{1}{\sin^2\pi\eta\Gamma(3+2\eta)}$$
$$\times\left\{-\frac{1+2\eta}{\Gamma(3-2\eta)(1+\eta)(2+\eta)}+\frac{1+2\eta}{12(1+\eta)(2+\eta)\Gamma(1-2\eta)}\right.$$
$$\left.\times\left[1+\frac{(3+2\eta)(1+\eta)}{15}\right]\right\}$$

After some calculations of residues at points $\eta = 0, 1, 2, \ldots$ we get

$$A = -\frac{13}{45} + 4\left\{\frac{1}{12}\sum_{k=1}^{\infty}\frac{1}{4k(k+1)^2(2+k)}\left[1+\frac{(3+2k)(1+k)}{15}\right]\right.$$
$$\left. -\frac{1}{8}\sum_{k=2}^{\infty}\frac{1}{k(1-k^2)(1-2k)(1+k)(2+k)}\right\} \approx -0.27$$

12. CONCLUSION

In this paper we have proposed a scheme for constructing a gauge-invariant nonlocal theory of four-fermion weak and electromagnetic interactions with the use of notion of the stochastic space. The main attention was paid to the investigations of the low-energy processes and to the proof of gauge invariance for the S matrix in each order of the perturbation series. In our case the S matrix obtained by the hypothesis of the stochastic space satisfies the fundamental principles of quantum field theory: Lorentz covariance, unitarity, causality, and gauge invariance.

The nonlocal corrections to the AMM of leptons and to the Lamb shift are calculated, and restrictions on the parameter of nonlocality (the elementary length l) are obtained. Also some consequences of neutrino oscillations and the electromagnetic properties of neutrinos are considered.

We believe that within our scheme all low-energy electromagnetic and "weak" processes may be described without contradiction with the experimental data. In our model, apart from the elementary length there exists a functional arbitrariness connected with the choice of a form of weak and electromagnetic potentials at small distances. This situation allows us to interpret our approach as a phenomenological scheme having unknown parameters in the theory. Therefore our model belongs to the second-class approaches mentioned in Section 1 of this review. In our case the occurrence of form factors in the theory, i.e., violation of the concept of locality at small distances, is connected with the stochasticity of space on small scale. Averaging of any fields independent of their nature (i.e., mass, spin, charge, etc.) over this stochastic space leads to the nonlocal fields considered by Efimov (1977). In other words, stochasticity of space (after averaging over a large scale) as a self-memory makes the theory nonlocal.

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